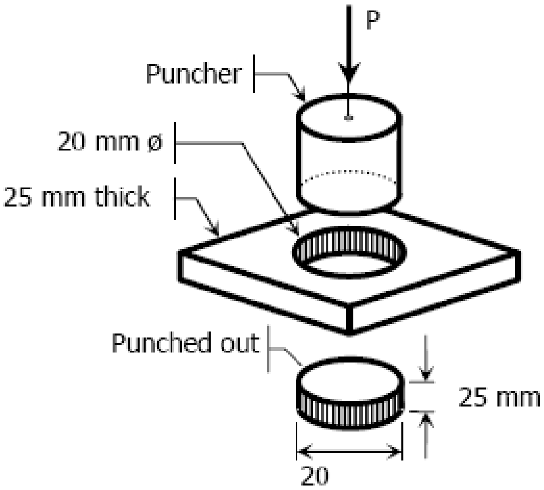
Strength of Materials 4th Edition by Pytel and Singer Problem 115 page 16

Given
Required diameter of hole = 20 mm
Thickne:ss of plate = 25 mm
Shear strength of plate = 350 MN/m^2

Required: Force required to punch a 20-mm-diameter hole

Solution 115



The resisting area is the shaded area along the perimeter and the shear force V is equal to the punching force P.

$$V = \tau A$$
$$P = 350\,[\,\pi(20)(25)\,]$$
$$P = 549\,778.7\ \text{N}$$
answer $P = 549.8\ \text{kN}$ →

Problem 117 page 17

Given:

Force P = 400 kN

Shear strength of the bolt = 300 MPa

The figure below:

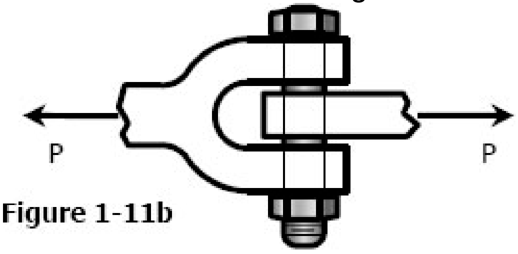


Figure 1-11b

Required: Diameter of the smallest bolt

Solution 117

The bolt is subject to double shear.

$$V = \tau A$$

$$400(1000) = 300[\,2(\tfrac{1}{4}\pi d^2)\,]$$

answer $d = 29.13\text{ mm} \;\rightarrow$

Problem 118 page 17

Given:

Diameter of pulley = 200 mm

Diameter of shaft = 60 mm

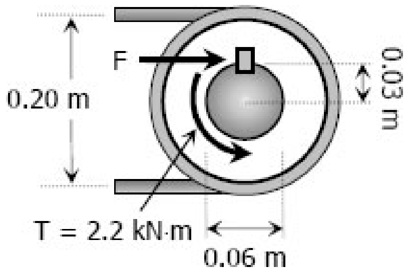
Length of key = 70 mm

Applied torque to the shaft = 2.5 kN·m

Allowable shearing stress in the key = 60 MPa

Required: Width b of the key

Solution 118

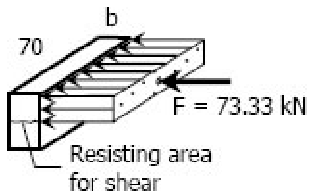


$$T = 0.03F$$

$$2.2 = 0.03F$$

$$F = 73.33\text{ kN}$$

$$V = \tau A$$



FBD of Pin

Where:

$V = F = 73.33\,\text{kN}$

$A = 70b$

$\tau = 60\,\text{MPa}$

$73.33(1000) = 60(70b)$

answer $b = 17.46\,\text{mm} \rightarrow$

Problem 119 page 17

Given:
Diameter of pin at B = 20 mm

Required: Shearing stress of the pin at B

Solution 119

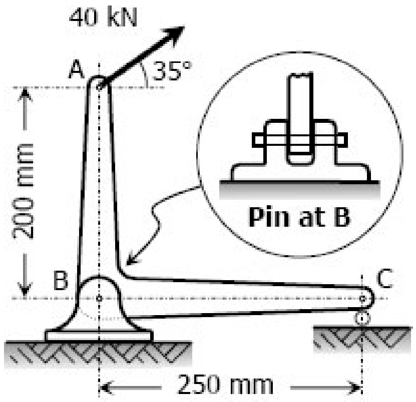


Figure P-119

From the FBD:

$\Sigma M_C = 0$

$0.25 R_{BV} = 0.25(40 \sin 35°) + 0.2(40 \cos 35°)$

$R_{BV} = 49.156\,\text{kN}$

$\Sigma F_H = 0$

$R_{BH} = 40 \cos 35°$

$R_{BH} = 32.766\,\text{kN}$

$R_B = \sqrt{R_{BH}^2 + R_{BV}^2}$

$R_B = \sqrt{32.766^2 + 49.156^2}$

shear force of pin at B $R_B = 59.076\,\text{kN} \rightarrow$

double shear $V_B = \tau_B\, A \rightarrow$

$59.076(1000) = \tau_B \left\{ 2\left[\frac{1}{4}\pi(20^2) \right] \right\}$

$\tau_B = 94.02\,\text{MPa} \rightarrow$

Problem 122 page 18

Given:

Width of wood = w

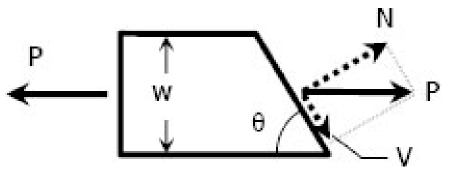
Thickness of wood = t

Angle of Inclination of glued joint = θ

Cross sectional area = A

Required: Show that shearing stress on glued joint $\tau = P \sin 2\theta \,/\, 2A$

Solution 122

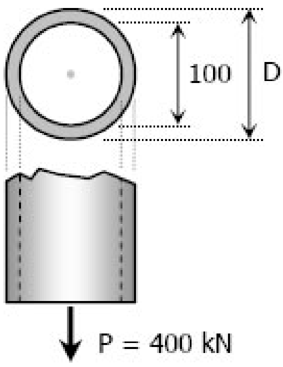


Shear area, $A_{\text{shear}} = t(w\csc\theta)$
Shear area, $A_{\text{shear}} = tw\csc\theta$
Shear area, $A_{\text{shear}} = A\ \csc\theta$
Shear force, $V = P\cos\theta$

$$V = \tau A_{\text{shear}}$$
$$P\cos\theta = \tau(A\csc\theta)$$
$$\tau = \frac{P\sin\theta\ \cos\theta}{A}$$
$$\tau = \frac{P(2\sin\theta\ \cos\theta)}{2A}$$
$$\tau = P\sin 2\theta\ /\ 2A$$

Problem 104

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.



$P = \sigma\,A$
$P = \sigma A$

where:

$P = 400\,\text{kN} = 400\,000\text{N}$
$\sigma = 120\,\text{MPa}$
$A = \frac{1}{4}\pi D^2 - \frac{1}{4}\pi(100^2)$
$A = \frac{1}{4}\pi(D^2 - 10\,000)$

thus,

$400\,000 = 120\,[\,\frac{1}{4}\pi(D^2 - 10\,000)\,]$

$400\,000 = 30\pi D^2 - 300\,000\pi$

$$D^2 = \frac{400\,000 + 300\,000\pi}{30\pi}$$

$D = 119.35\ \mathrm{mm}\ \rightarrow$

Problem 105 page 12

Given:

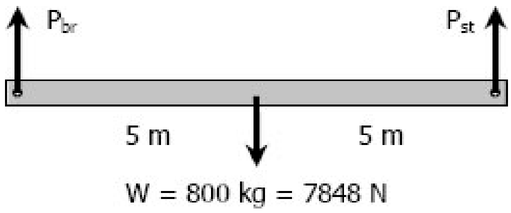
Weight of bar = 800 kg

Maximum allowable stress for bronze = 90 MPa

Maximum allowable stress for steel = 120 MPa

Required: Smallest area of bronze and steel cables

Solution 105



By symmetry:

$$P_{br} = P_{st} = \frac{1}{2}(7848)$$
$$P_{br} = 3924 N$$
$$P_{st} = 3924 N$$

For bronze cable:

$$P_{br} = \sigma_{br} A_{br}$$
$$3924 = 90\, A_{br}$$

answer $A_{br} = 43.6\ \mathrm{mm}^2\ \rightarrow$

For steel cable:

$$P_{st} = \sigma_{st}\, A_{st}$$
$$3924 = 120\, A_{st}$$
$$A_{st} = 32.7\ \mathrm{mm}^2\ \rightarrow$$

Problem 108 page 12

Given:

Maximum allowable stress for steel = 140 MPa

Maximum allowable stress for aluminum = 90 MPa

Maximum allowable stress for bronze = 100 MPa

Required: Maximum safe value of axial load P

Solution 108

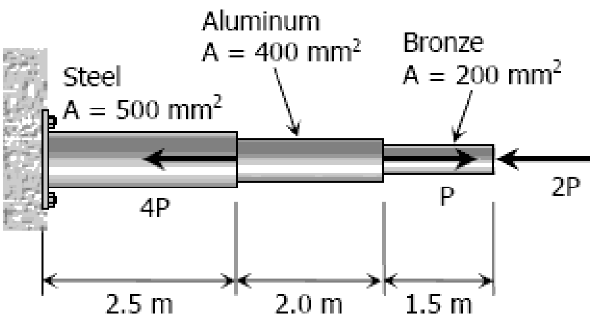


Figure P-108

For bronze:

$$\sigma_{br} A_{br} = 2P$$

$$100(200) = 2P$$
$$P = 10\,000\,\text{N}$$

For aluminum:

$$\sigma_{al}A_{al} = P$$
$$90(400) = P$$
$$P = 36\,000\,\text{N}$$

For Steel:

$$\sigma_{st}A_{st} = 5P$$
$$P = 14\,000\,\text{N}$$

For safe P, use $P = 10\,000\,\text{N} = 10\,\text{kN}$ $\rightarrow$***answer***

Problem 125

In Fig. 1-12, assume that a 20-mm-diameter rivet joins the plates that are each 110 mm wide. The allowable stresses are 120 MPa for bearing in the plate material and 60 MPa for shearing of rivet. Determine (a) the minimum thickness of each plate; and (b) the largest average tensile stress in the plates.

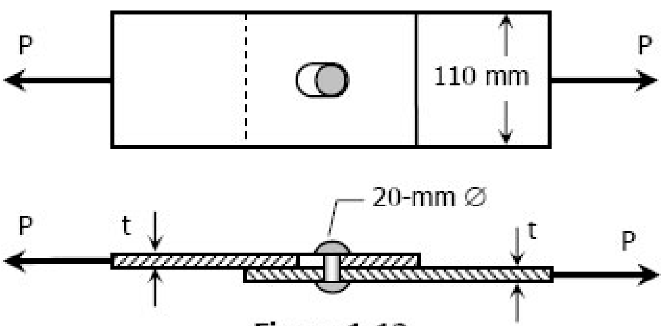


Figure 1-12

Solution 125

Part (a):

From shearing of rivet:

$$P = \tau A_{rivets}$$
$$P = 60[\,\tfrac{1}{4}\pi(20^2)\,]$$
$$P = 6000\pi\ textN$$

From bearing of plate material:

$$P = \sigma_b A_b$$
$$6000\pi = 120(20t)$$

answer $t = 7.85\text{ mm}$ $\rightarrow$

Part (b): Largest average tensile stress in the plate:

$$P = \sigma A$$
$$6000\pi = \sigma[\,7.85(110 - 20)\,]$$

answer $\sigma = 26.67\text{ MPa}$ $\rightarrow$

Problem 129 page 21

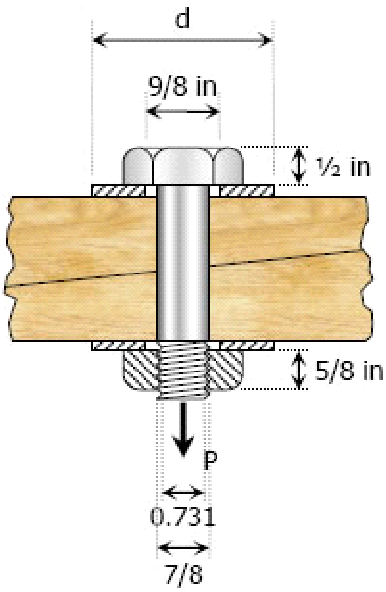
Given:

Diameter of bolt = 7/8 inch
Diameter at the root of the thread (bolt) = 0.731 inch
Inside diameter of washer = 9/8 inch
Tensile stress in the nut = 18 ksi
Bearing stress = 800 psi

Required:

Shearing stress in the head of the bolt
Shearing stress in threads of the bolt
Outside diameter of the washer

Solution 129



Tensile force on the bolt:

$$P = \sigma A = 18[\,\tfrac{1}{4}\pi(\tfrac{7}{8})^2\,]$$

$$P = 10.82\,\text{kips}$$

Shearing stress in the head of the bolt:

$$\tau = \frac{P}{A} = \frac{10.82}{\pi(\frac{7}{8})(\frac{1}{2})}$$

answer $\tau = 7.872\,\text{ksi} \;\rightarrow$

Shearing stress in the threads:

$$\tau = \frac{P}{A} = \frac{10.82}{\pi(0.731)(\frac{5}{8})}$$

answer $\tau = 7.538\,\text{ksi} \;\rightarrow$

Outside diameter of washer:

$$P = \sigma_b\,A_b$$

$$10.82(1000) = 800\{\,\tfrac{1}{4}\pi\,[\,d^2 - (\tfrac{9}{8})^2\,]\,\}$$

and $d = 4.3\,\text{inch} \;\rightarrow$

Problem 130 page 22

Given:

Allowable shear stress = 70 MPa
Allowable bearing stress = 140 MPa
Diameter of rivets = 19 mm
The truss below:

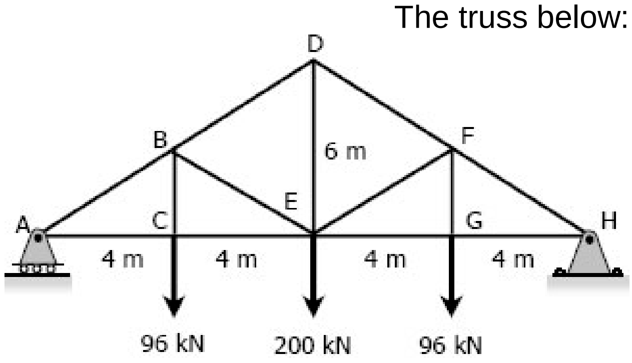


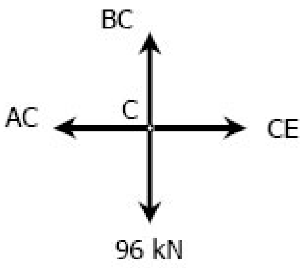
Figure P-130 and P-131

Required:

Number of rivets to fasten member BC to the gusset plate
Number of rivets to fasten member BE to the gusset plate
Largest average tensile or compressive stress in members BC and BE

Solution 130

At Joint C:

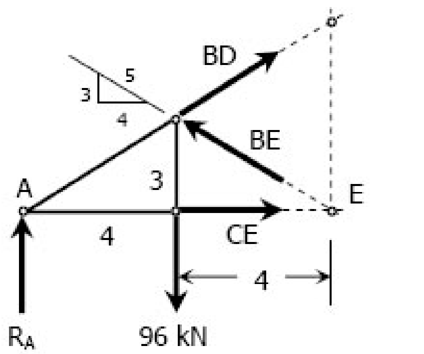


Joint C

$$\Sigma F_V = 0$$

(Tension) $BC = 96\text{ kN}$

Consider the section through member BD, BE, and CE:



Section through BD, BE, and CE

$$\Sigma M_A = 0$$

$$8(\tfrac{3}{5}BE) = 4(96)$$

(Compression) $BE = 80\text{ kN}$

For Member BC:

Based on shearing of rivets:

Where A = area of 1 rivet × number of rivets, n $BC = \tau A$

$96\,000 = 70[\,\frac{1}{4}\pi(19^2)n\,]$

say 5 rivets $n = 4.8$

Based on bearing of member:

$BC = \sigma_b\,A_b$

Where A_b = diameter of rivet × thickness of BC × number of rivets, n

$96\,000 = 140[\,19(6)n\,]$

say 7 rivets $n = 6.02$

use 7 rivets for member BC ***answer***

For member BE:

Based on shearing of rivets:

$BE = \tau\,A$

Where A = area of 1 rivet × number of rivets, n

$80\,000 = 70[\,\frac{1}{4}\pi(19^2)n\,]$

say 5 rivets $n = 4.03$

Based on bearing of member:

$BE = \sigma_b\,A_b$

Where A_b = diameter of rivet × thickness of BE × number of rivets, n

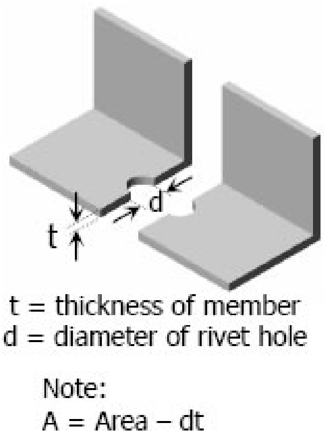
$80\,000 = 140[\,19(13)n\,]$

say 3 rivets $n = 2.3$

use 5 rivets for member BE ***answer***

Relevant data from the table (Appendix B of textbook): *Properties of Equal Angle Sections: SI Units*

Designation	Area
L75 × 75 × 6	864 mm^2
L75 × 75 × 13	1780 mm^2



Tensile stress of member BC (L75 × 75 × 6):

$$\sigma = \frac{P}{A} = \frac{96(1000)}{864 - 19(6)}$$

answer $\sigma = 128\ \text{Mpa}\ \rightarrow$

Compressive stress of member BE (L75 × 75 × 13):

$$\sigma = \frac{P}{A} = \frac{80(1000)}{1780}$$

answer $\sigma = 44.94\ \text{Mpa}\ \rightarrow$

Problem 131

Repeat Problem 130 if the rivet diameter is 22 mm and all other data remain unchanged.

Solution 131

For member BC:

(Tension) $P = 96\ \text{kN}$

Based on shearing of rivets:

$$P = \tau\,A$$

$$96\,000 = 70[\,\tfrac{1}{4}\pi(22^2)n\,]$$

say 4 rivets $n = 3.6$

Based on bearing of member:

$$P = \sigma_b\,A_b$$

$$96\,000 = 140[\,22(6)n\,]$$

say 6 rivets $n = 5.2$

Use 6 rivets for member BC ***answer***

Tensile stress:

$$\sigma = \frac{P}{A} = \frac{96(1000)}{864 - 22(6)}$$

answer $\sigma = 131.15\ \text{MPa}\ \rightarrow$

For member BE:

(Compression) $P = 80\ \text{kN}$

Based on shearing of rivets:

$$P = \tau\,A$$

$$80\,000 = 70[\,\tfrac{1}{4}\pi(22^2)n\,]$$

say 4 rivets $n = 3.01$

Based on bearing of member:

$$P = \sigma_b\,A_b$$

$$80\,000 = 140[\,22(13)n\,]$$

say 2 rivets $n = 1.998$

use 4 rivets for member BE ***answer***

Compressive stress:

$$\sigma = \frac{P}{A} = \frac{80(1000)}{1780}$$

answer $\sigma = 44.94 \text{ MPa} \rightarrow$

Problem 136 page 28

Given:

Thickness of steel plating = 20 mm

Diameter of pressure vessel = 450 mm

Length of pressure vessel = 2.0 m

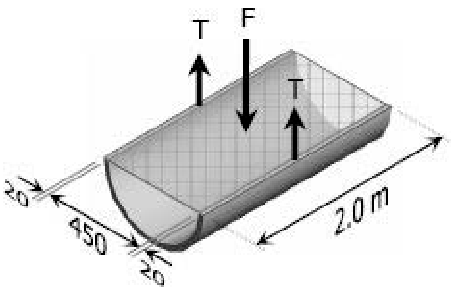
Maximum longitudinal stress = 140 MPa

Maximum circumferential stress = 60 MPa

Required: The maximum internal pressure that can be applied

Solution 136

Based on circumferential stress (tangential):



$$\Sigma F_V = 0$$

$$F = 2T$$

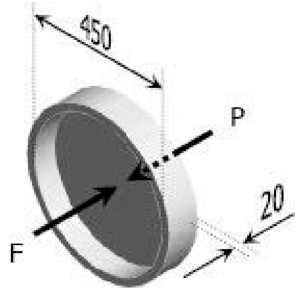
$$p(DL) = 2(\sigma_t L_t)$$

$$\sigma_t = \frac{pD}{2t}$$

$$60 = \frac{p(450)}{2(20)}$$

$$p = 5.33 \text{ MPa}$$

Based on longitudinal stress:



$$\Sigma F_H = 0$$
$$F = P$$
$$p(\tfrac{1}{4}\pi D^2) = \sigma_l(\pi D)$$
$$\sigma_l = \frac{pD}{4t}$$
$$140 = \frac{p(450)}{4(20)}$$
$$p = 24.89\ \mathrm{MPa}$$

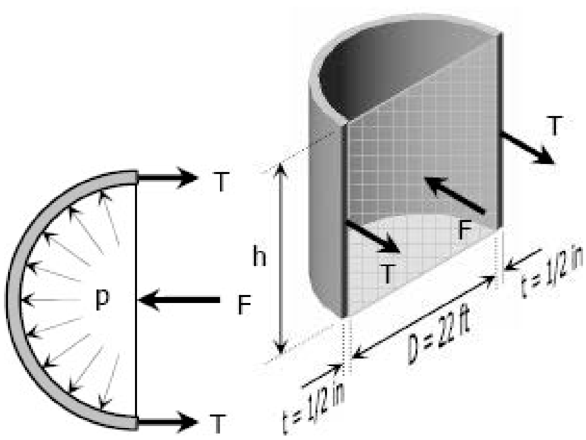
Use $p = 5.33\ \mathrm{MPa}$ →***answer***

Problem 137 page 28

Given:
Diameter of the water tank = 22 ft
Thickness of steel plate = 1/2 inch
Maximum circumferential stress = 6000 psi
Specific weight of water = 62.4 lb/ft^3

Required: The maximum height to which the tank may be filled with water.

Solution 137



$$\sigma_t = 6000\ \mathrm{psi} = 6000\ \mathrm{lb/in^2}\,(12\ \mathrm{in/ft})^2$$
$$\sigma_t = 864\,000\ \mathrm{lb/ft^2}$$

Assuming pressure distribution to be uniform:

$$p = \gamma h = 62.4h$$

$$F = pA = 62.4h(Dh)$$
$$F = 62.4(22)h^2$$
$$F = 1372.8h^2$$

$$T = \sigma_t\,A_t = 864\,000(th)$$
$$T = 864\,000(\tfrac{1}{2} \times \tfrac{1}{12})\,h$$
$$T = 36\,000h$$
$$\Sigma F = 0$$
$$F = 2T$$
$$1372.8h^2 = 2(36\,000h)$$
answer $h = 52.45\ \text{ft}\ \rightarrow$

Problem 139 page 28

Given:

Allowable stress = 20 ksi

Weight of steel = 490 lb/ft^3

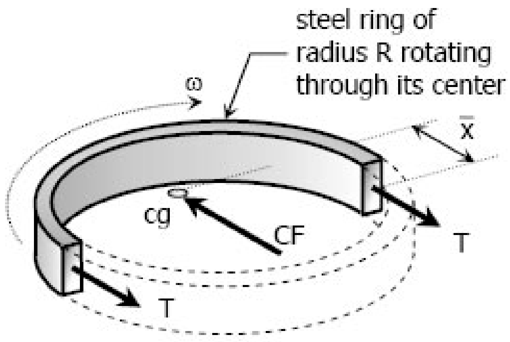
Mean radius of the ring = 10 inches

Required:

The limiting peripheral velocity.

The number of revolution per minute for stress to reach 30 ksi.

Solution 139



FBD of Ring in Rotation

Centrifugal Force, CF:

$$CF = M\,\omega^2\,\bar{x}$$

where:

$$M = \frac{W}{g}\ \frac{\gamma V}{g} = \frac{\gamma \pi R A}{g}$$
$$\omega = v/R$$
$$\bar{x} = 2R/\pi$$
$$CF = \frac{\gamma \pi R A}{g}\left(\frac{v}{R}\right)^2\left(\frac{2R}{\pi}\right)$$
$$CF = \frac{2\gamma A v^2}{g}$$
$$2T = CF$$
$$2\gamma A = \frac{2\gamma A v^2}{g}$$
$$\sigma = \frac{\gamma\,v^2}{g}$$

From the given data:

$$\sigma = 20\,\text{ksi} = (20\,000\,\text{lb/in}^2)(12\,\text{in/ft})a^2$$

$$\sigma = 2\,880\,000\,\text{lb/ft}^2$$
$$\gamma = 490\,\text{lb/ft}^3$$
$$2\,880\,000 = \frac{490v^2}{32.2}$$
answer $v = 435.04\ \text{ft/sec} \rightarrow$

When $\sigma = 30\ \text{ksi}$, and $R = 10\ \text{in}$
$$\sigma = \frac{\gamma\, v^2}{g}$$
$$30\,000(12^2) = \frac{490v^2}{32.2}$$
$$v = 532.81\ \text{ft/sec}$$
$$\omega = v/R = \frac{532.81}{10/12}$$
$$\omega = 639.37\ \text{rad/sec}$$
$$\omega = \frac{639.37\ \text{rad}}{\text{sec}} \times \frac{1\ \text{rev}}{2\pi\ \text{rad}} \times \frac{60\ \text{sec}}{1\ \text{min}}$$
answer $\omega = 6,105.54\ \text{rpm} \rightarrow$

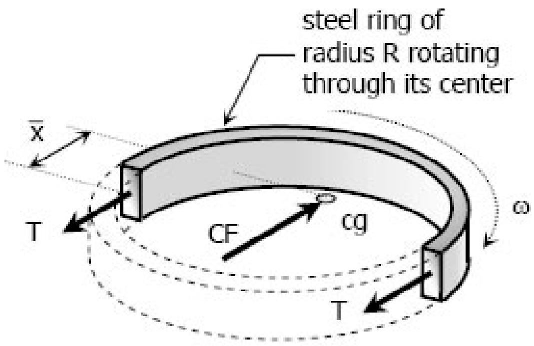
Problem 140 page 28

Given:

Stress in rotating steel ring = 150 MPa
Mean radius of the ring = 220 mm
Density of steel = 7.85 Mg/m^3

Required: Angular velocity of the steel ring

Solution 140



FBD of Ring in Rotation

$$CF = M\omega^2\bar{x}$$

Where:
$$M = \rho V = \rho A \pi R$$
$$x = 2R/\pi$$

$$CF = \rho A \pi R \omega^2 (2R/\pi)$$
$$CF = 2\rho A R^2 \omega^2$$

$$2T = CF$$
$$2\sigma A = 2\rho A R^2 \omega^2$$
$$\sigma = \rho R^2 \omega^2$$

From the given (Note: 1 N = 1 kg·m/sec^2):

$$\sigma = 150\,\text{MPa}$$
$$\sigma = 150\,000\,000\,\text{kg}\cdot\text{m/sec}^2\cdot\text{m}^2$$
$$\sigma = 150\,000\,000\,\text{kg/m}\cdot\text{sec}^2$$
$$\rho = 7.85\,\text{Mg/m}^3 = 7850\,\text{kg/m}^3$$
$$R = 220\,\text{mm} = 0.22\,\text{m}$$
$$150\,000\,000 = 7850(0.22)^2\,\omega^2$$

answer $\omega = 628.33\,\text{rad/sec} \;\rightarrow$

Problem 142 page 29

Given:

Steam pressure = 3.5 Mpa

Outside diameter of the pipe = 450 mm

Wall thickness of the pipe = 10 mm

Diameter of the bolt = 40 mm

Allowable stress of the bolt = 80 MPa

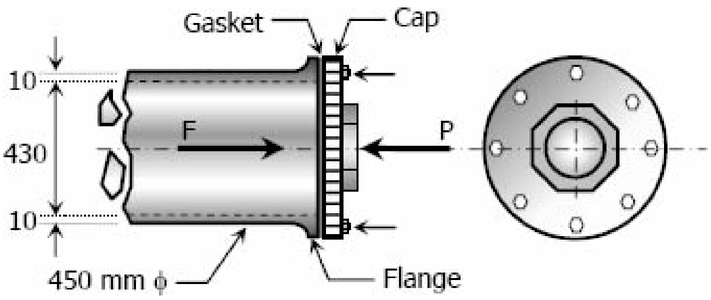
Initial stress of the bolt = 50 MPa

Required:

Number of bolts

Circumferential stress developed in the pipe

Solution 29

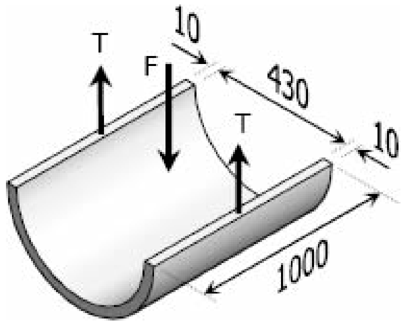


$$F = \sigma A$$
$$F = 3.5[\,\tfrac{1}{4}\pi(430^2)\,]$$
$$F = 508\,270.42\,\text{N}$$

$$P = F$$
$$(\sigma_{\text{bolt}} A)\,n = 508\,270.42\,\text{N}$$
$$(80 - 55)[\,\tfrac{1}{4}\pi(40^2)\,]n = 508\,270.42$$

say 17 bolts $\rightarrow$ ***answer*** $n = 16.19$

Circumferential stress (consider 1-m strip):



$$F = pA = 3.5[\,430(1000)\,]$$
$$F = 1\,505\,000\ \mathrm{N}$$

$$2T = F$$
$$2[\,\sigma_t(1000)(10)\,] = 1\,505\,000$$
answer $\sigma_t = 75.25\ \mathrm{MPa}\ \rightarrow$

Discussion:

It is necessary to tighten the bolts initially to press the gasket to the flange, to avoid leakage of steam. If the pressure will cause 110 MPa of stress to each bolt causing it to fail, leakage will occur. If this is sudden, the cap may blow.

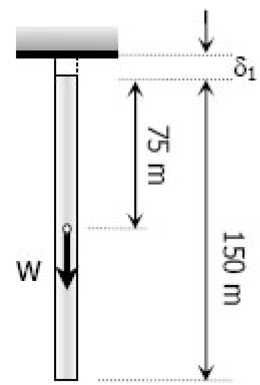
Problem 206 page 39

Given:
Cross-sectional area = 300 mm^2
Length = 150 m
tensile load at the lower end = 20 kN
Unit mass of steel = 7850 kg/m^3
E = 200 × 10^3 MN/m^2

Required: Total elongation of the rod

Solution 206

Elongation due to its own weight:



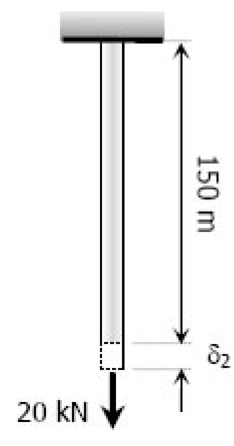
$\delta_1 = \dfrac{PL}{AE}$

Where:
P = W = 7850(1/1000)^3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$\delta_1 = \dfrac{3\,465.3825(75\,000)}{300(200\,000)}$

δ_1= 4.33 mm

Elongation due to applied load:



$\delta_2 = \dfrac{PL}{AE}$

Where:
P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2
E = 200 000 MPa

$\delta_2 = \dfrac{20\,000(150\,000)}{300(200\,000)}$

δ_2= 50 mm

Total elongation:

$\delta = \delta_1 + \delta_2$

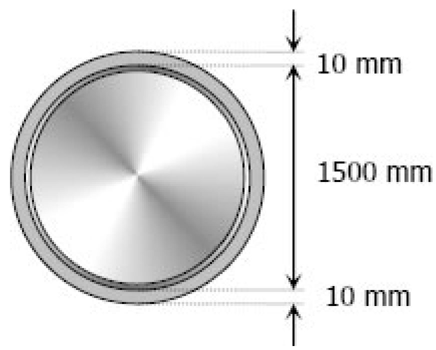
$\delta = 4.33 + 50 = 54.33\text{ mm}$ →***answer***

Problem 208 page 40

Given:
Thickness of steel tire = 100 mm
Width of steel tire = 80 mm
Inside diameter of steel tire = 1500.0 mm
Diameter of steel wheel = 1500.5 mm
Coefficient of static friction = 0.30
E = 200 GPa

Required: Torque to twist the tire relative to the wheel

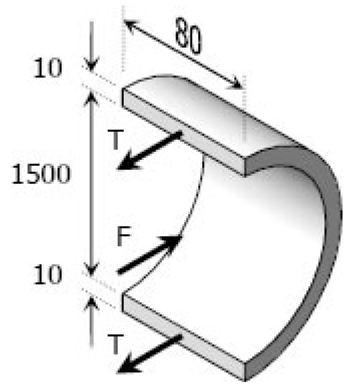
Solution 208



$$\delta = \frac{PL}{AE}$$

Where:
$\delta = \pi$ (1500.5 - 1500) = 0.5π mm
P = T
L = 1500π mm
A = 10(80) = 800 mm^2
E = 200 000 MPa

$$0.5\pi = \frac{T(1500\pi)}{800(200\,000)}$$
$$T = 53\,333.33\ \text{N}$$



$F = 2T$
$p(1500)(80) = 2(53\,333.33)$
$p = 0.8889\text{ MPa}$ →internal pressure

Total normal force, N:
N = p × contact area between tire and wheel
N = 0.8889 × π(1500.5)(80)
N = 335 214.92 N

Friction resistance, f:
f = μN = 0.30(335 214.92)
f = 100 564.48 N = 100.56 kN
Torque = f × ½(diameter of wheel)
Torque = 100.56 × 0.75025
Torque = **75.44 kN · m**

Problem 211 page 40

Given:
Maximum overall deformation = 3.0 mm
Maximum allowable stress for steel = 140 MPa
Maximum allowable stress for bronze = 120 MPa
Maximum allowable stress for aluminum = 80 MPa
E_{st} = 200 GPa
E_{al} = 70 GPa
E_{br} = 83 GPa

The figure below:

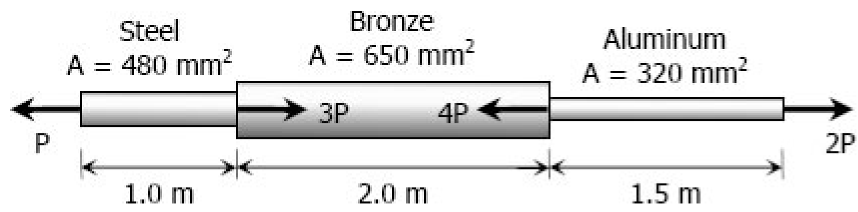
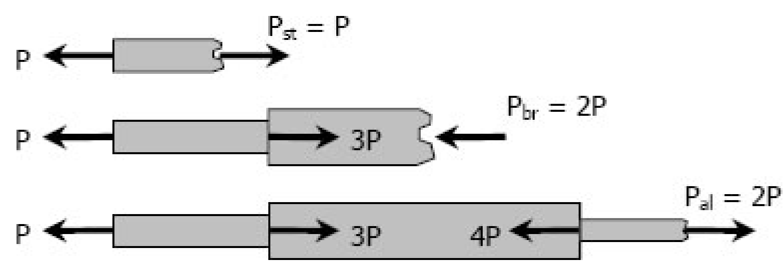


Figure P -211

Required: The largest value of P

Solution 211



Based on allowable stresses:

Steel:

$$P_{st} = \sigma_{st}\, A_{st}$$
$$P = 140(480) = 67\,200\ \text{N}$$
$$P = 67.2\ \text{kN}$$

Bronze:

$$P_{br} = \sigma_{br}\, A_{br}$$
$$2P = 120(650) = 78\,000\ \text{N}$$
$$P = 39\,000\ \text{N} = 39\ \text{kN}$$

Aluminum:

$$P_{al} = \sigma_{al} A_{al}$$
$$2P = 80(320) = 25\,600 \text{ N}$$
$$P = 12\,800 \text{ N} = 12.8 \text{ kN}$$

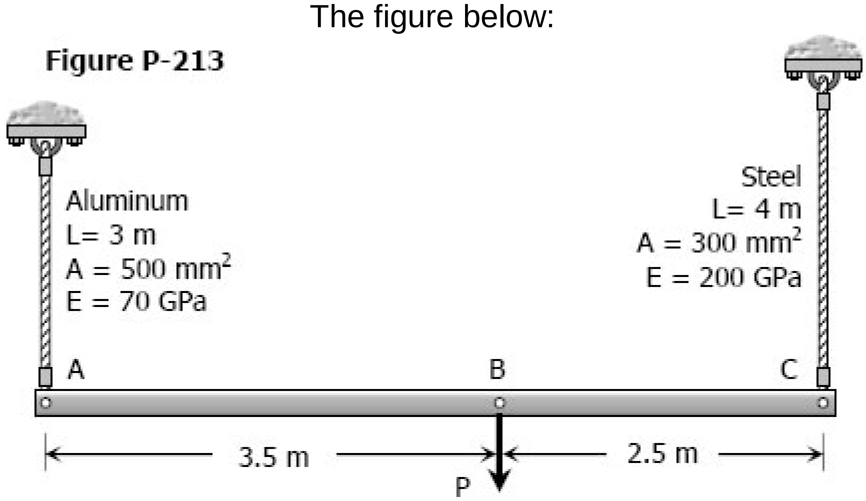
Based on allowable deformation:
(*steel and aluminum lengthens, bronze shortens*)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$
$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70\,000)} + \frac{2P(1500)}{320(83\,000)}$$
$$3 = \left(\frac{1}{96\,000} - \frac{1}{11\,375} + \frac{3}{26\,560}\right) P$$
$$P = 84\,610.99 \text{ N} = 84.61 \text{ kN}$$

Use the smallest value of P, **P = 12.8 kN**

Problem 213 page 41

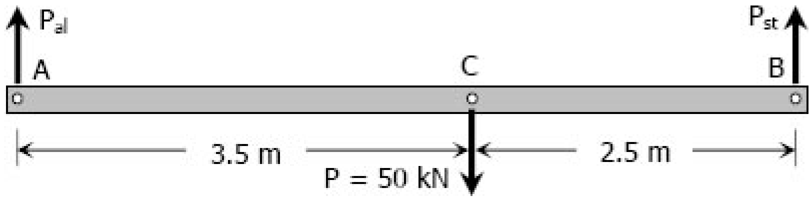
Given:
Rigid bar is horizontal before P = 50 kN is applied
The figure below:



Required: Vertical movement of P

Solution 213

Free body diagram:



For aluminum:

$\Sigma M_B = 0$
$6P_{al} = 2.5(50)$
$P_{al} = 20.83\,\text{kN}$

$$\delta = \frac{PL}{AE}$$

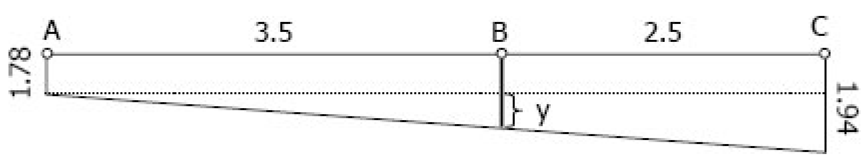
$$\delta_{al} = \frac{20.83(3)1000^2}{500(70\,000)}$$
$\delta_{al} = 1.78\,\text{mm}$

For steel:

$\Sigma M_A = 0$
$6P_{st} = 3.5(50)$
$P_{st} = 29.17\,\text{kN}$

$$\delta = \frac{PL}{AE}$$
$$\delta_{st} = \frac{29.17(4)1000^2}{300(200\,000)}$$
$\delta_{st} = 1.94\,\text{mm}$

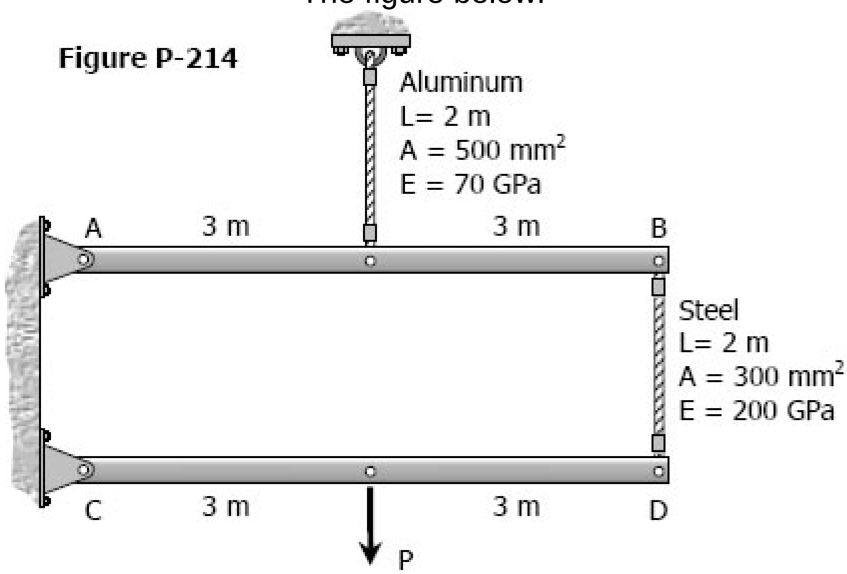
Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$
$y = 0.09\,\text{mm}$
$\delta_B = \text{ vertical movement of } P$
$\delta_B = 1.78 + y = 1.78 + 0.09$
$\delta_B = 1.87\,\text{mm}$ →***answer***

Problem 214 page 41

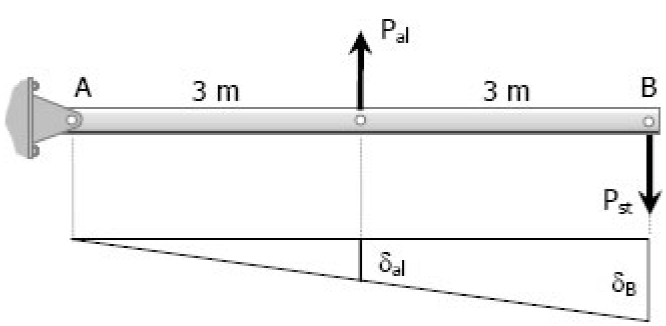
Given:
Maximum vertical movement of P = 5 mm
The figure below:



Required: The maximum force P that can be applied neglecting the weight of all members.

Solution 41

Member AB:

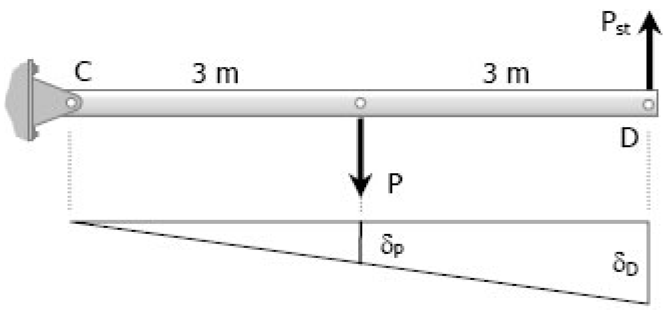


FBD and movement diagram of bar AB

$\Sigma M_A = 0$
$3P_{al} = 6P_{st}$
$P_{al} = 2P_{st}$

By ratio and proportion:
$\dfrac{\delta_B}{6} = \dfrac{\delta_{al}}{3}$
$\delta_B = 2\delta_{al} = 2\left[\dfrac{PL}{AE}\right]_{al}$
$\delta_B = 2\left[\dfrac{P_{al}\,(2000)}{500(70\,000)}\right]$
$\delta_B = \frac{1}{8750}P_{al} = \frac{1}{8750}(2P_{st})$
$\delta_B = \frac{1}{4375}P_{st}$ $\rightarrow$movement of B

Member CD:



FBD and movement diagram of bar CD

Movement of D:
$\delta_D = \delta_{st} + \delta_B = \left[\dfrac{PL}{AE}\right]_{st} + \frac{1}{4375}P_{st}$
$\delta_D = \dfrac{P_{st}\,(2000)}{300(200\,000)} + \frac{1}{4375}P_{st}$
$\delta_D = \frac{11}{42\,000}P_{st}$

$\Sigma M_C = 0$
$6P_{st} = 3P$
$P_{st} = \frac{1}{2}P$

By ratio and proportion:
$\dfrac{\delta_P}{3} = \dfrac{\delta_D}{6}$
$\delta_P = \frac{1}{2}\delta_D = \frac{1}{2}(\frac{11}{42\,000}P_{st})$
$\delta_P = \frac{11}{84\,000}P_{st}$
$5 = \frac{11}{84\,000}(\frac{1}{2}P)$
$P = 76\,363.64\ \mathrm{N} = 76.4\ \mathrm{kN}$ $\rightarrow$***answer***

Problem 225

A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

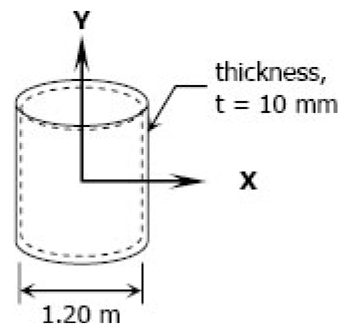
Solution 225

σ_y = longitudinal stress

$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$
$$\sigma_y = 45\,\text{MPa}$$

σ_x = tangential stress

$$\sigma_y = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$
$$\sigma_y = 90\,\text{MPa}$$



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$
$$\varepsilon_x = \frac{90}{200\,000} - 0.3\left(\frac{45}{200\,000}\right)$$
$$\varepsilon_x = 3.825 \times 10^{-4}$$

$$\varepsilon_x = \frac{\Delta D}{D}$$
$$\Delta D = \varepsilon_x\, D == (3.825 \times 10^{-4})(1200)$$
$$\Delta D = 0.459\,\text{mm}$$ →***answer***

Problem 227

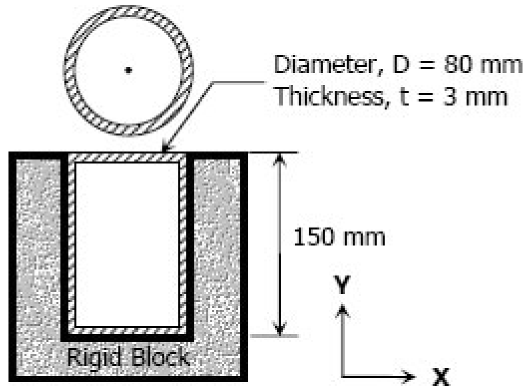
A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming ν = 1/3 and E = 83 GPa, determine the tangential stress in the tube.

Solution 227

Longitudinal stress:

$\sigma_y = \dfrac{pD}{4t} = \dfrac{4(80)}{4(3)}$

$\sigma_y = \frac{80}{3}\ \text{MPa}$



The **strain in the x-direction** is:

$\varepsilon_x = \dfrac{\sigma_x}{E} - \nu\dfrac{\sigma_y}{E} = 0$

$\sigma_x = \nu\,\sigma_y$ = tangential stress

$\sigma_x = \frac{1}{3}(\frac{80}{3})$

$\sigma_x = 8.89\ \text{MPa}$ →***answer***

Statically indeterminate

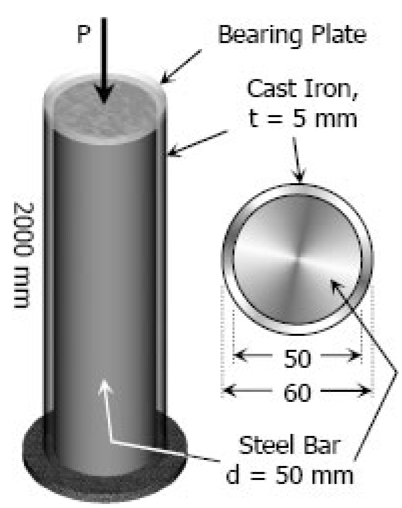
Problem 233

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233

$\delta = \dfrac{PL}{AE}$

$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8\ \text{mm}$



$\delta_{cast\ iron} = \dfrac{P_{cast\ iron}(2000)}{[\,\frac{1}{4}\pi(60^2 - 50^2)\,](100\,000)} = 0.8$

$P_{cast\ iron} = 11\,000\pi\ \text{N}$

$\delta_{steel} = \dfrac{P_{steel}(2000)}{[\frac{1}{4}\pi(50^2)\,](200\,000)} = 0.8$

$P_{steel} = 50\,000\pi\ \text{N}$

$\Sigma F_V = 0$

$P = P_{cast\ iron} + P_{steel}$

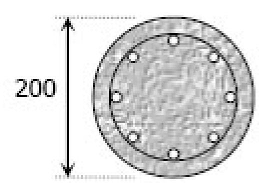
$$P = 11\,000\pi + 50\,000\pi$$
$$P = 61\,000\pi \text{ N}$$
$$P = 191.64 \text{ kN}$$ →***answer***

Problem 234

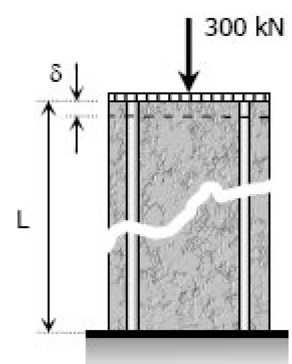
A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.

Solution 234

$$\delta_{co} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200\,000}$$
$$100\sigma_{co} = 7\sigma_{st}$$



When $\sigma_{st} = 120 \text{ MPa}$
$$100\sigma_{co} = 7(120)$$
$\sigma_{co} = 8.4 \text{ MPa} > 6 \text{ MPa}$ (not ok!)



When $\sigma_{co} = 6 \text{ MPa}$
$$100(6) = 7\sigma_{st}$$
$\sigma_{st} = 85.71 \text{ MPa} < 120 \text{ MPa}$ (ok!)

Use $\boldsymbol{\sigma_{co}}$ **= 6 MPa** and $\boldsymbol{\sigma_{st}}$ **= 85.71 MPa**

$$\Sigma F_V = 0$$
$$P_{st} + P_{co} = 300$$
$$\sigma_{st}\,A_{st} + \sigma_{co}\,A_{co} = 300$$
$$85.71Ast + 6\,[\,\tfrac{1}{4}\pi(200)^2 - A_{st}\,] = 300(1000)$$
$$79.71A_{st} + 60\,000\pi = 300\,000$$
$$A_{st} = 1398.9\text{ mm}^2 \rightarrow \textbf{\textit{answer}}$$

Problem 236

A rigid block of mass M is supported by three symmetrically spaced rods as shown in Fig. P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

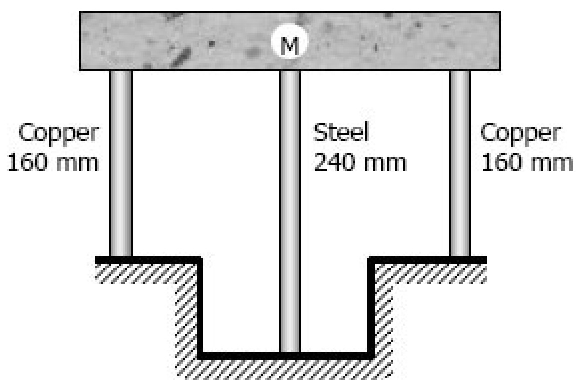
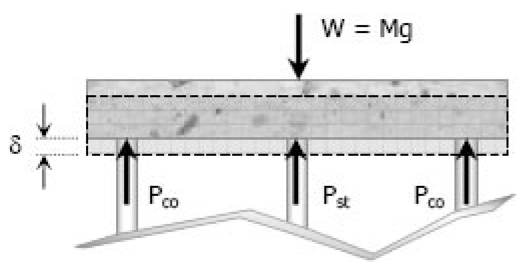


Figure P-236 and P-237

Solution 236



$$\delta_{co} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co}\,L}{120\,000} = \frac{\sigma_{st}\,L}{200\,000}$$
$$10\sigma_{co} = 9\sigma_{st}$$

When σ_{st} = 140 MPa

$\sigma_{co} = \frac{9}{10}(140)$

$\sigma_{co} = 126 \text{ MPa} > 70 \text{ MPa}$(**not ok!**)

When σ_{co} = 70 MPa

$\sigma_{st} = \frac{10}{9}(70)$

$\sigma_{st} = 77.78 \text{ MPa} < 140 \text{ MPa}$(**ok!**)

Use **σ_{co} = 70 MPa** and **σ_{st} = 77.78 MPa**

$$\Sigma F_V = 0$$
$$2P_{co} + P_{st} = W$$
$$2(\sigma_{co} A_{co}) + \sigma_{st} A_{st} = Mg$$
$$2[\,70(900)\,] + 77.78(1200) = M(9.81)$$
$$M = 22358.4 \text{ kg}$$ → ***answer***

Problem 239

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm2 and E = 70 GPa.

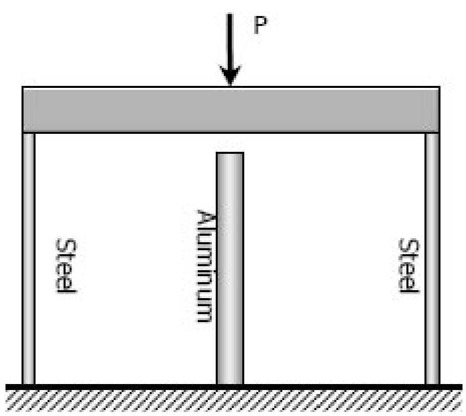
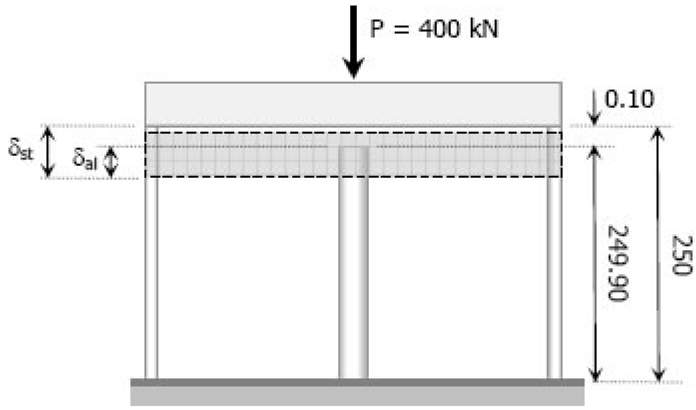
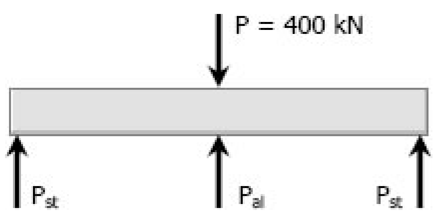


Figure P-239

Solution 239



$$\delta_{st} = \delta_{al} + 0.10$$
$$\left(\frac{\sigma L}{E}\right)_{st} = \left(\frac{\sigma L}{E}\right)_{al} + 0.10$$
$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$
$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$
$$\sigma_{st} = 2.856\sigma_{al} + 80$$



$$\Sigma F_V = 0$$
$$2P_{st} + P_{al} = 400\,000$$
$$2\sigma_{st}\,A_{st} + \sigma_{al}\,A_{al} = 400\,000$$
$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}(2400) = 400\,000$$
$$9254.4\sigma_{al} + 192\,000 = 400000$$
$\sigma_{al} = 22.48\ \text{MPa}$ → ***answer***

Problem 242

The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

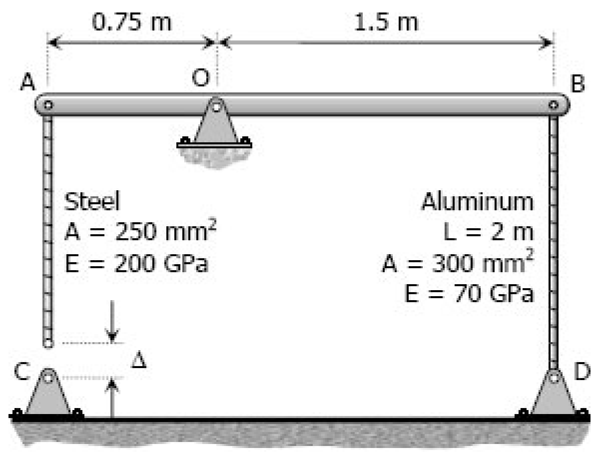
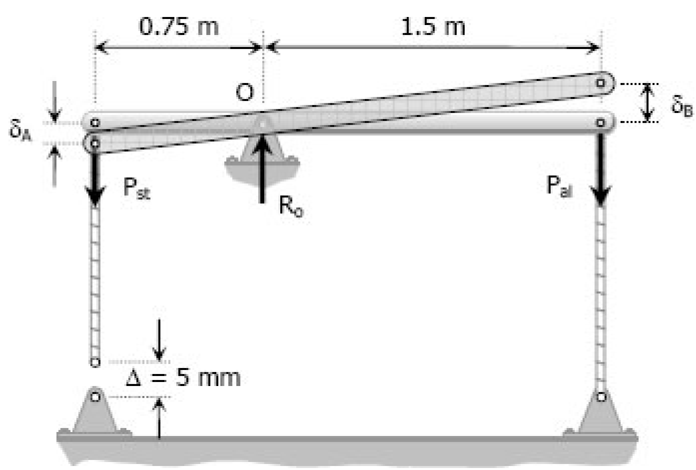


Figure P-242

Solution 242



$\Sigma M_O = 0$
$0.75P_{st} = 1.5P_{al}$
$P_{st} = 2P_{al}$
$\sigma_{st}\,A_{st} = 2(\sigma_{al}A_{al})$

$$\sigma_{st} = \frac{2\sigma_{al}\,A_{al}}{A_{st}}$$
$$\sigma_{st} = \frac{2\,[\,\sigma_{al}(3000\,]}{250}$$
$$\sigma_{st} = 2.4\sigma_{al}$$

$$\delta_{al} = \delta_B$$

By ratio and proportion:

$$\frac{\delta_A}{0.75} = \frac{\delta_B}{1.5}$$
$$\delta_A = 0.5\delta_B$$
$$\delta_A = 0.5\delta_{al}$$

$$\Delta = \delta_{st} + \delta_A$$
$$5 = \delta_{st} + 0.5\delta_{al}$$

$$5 = \frac{\sigma_{st}(2\,000 - 5)}{250(200\,000)} + 0.5\left[\frac{\sigma_{al}(2000)}{300(70\,000)}\right]$$

$$5 = (3.99\times10^{-5})\,\sigma_{st} + (4.76\times10^{-5})\,\sigma_{al}$$
$$\sigma_{al} = 105\,000 - 0.8379\,\sigma_{st}$$
$$\sigma_{al} = 105\,000 - 0.8379(2.4\,\sigma_{al})$$
$$3.01096\sigma_{al} = 105\,000$$
$$\sigma_{al} = 34\,872.6\ \text{MPa}$$ →***answer***

Problem 244

A homogeneous bar with a cross sectional area of 500 mm^2 is attached to rigid supports. It carries the axial loads P_1 = 25 kN and P_2 = 50 kN, applied as shown in Fig. P-244. Determine the stress in segment BC. (Hint: Use the results of Prob. 243, and compute the reactions caused by P_1 and P_2 acting separately. Then use the principle of superposition to compute the reactions when both loads are applied.)

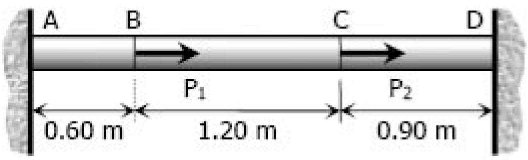
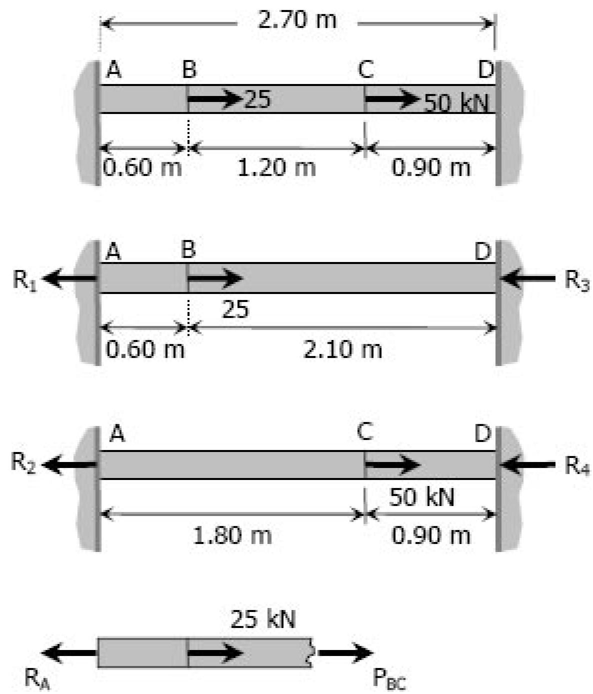


Figure P-244

Solution 244



From the results of [Solution to Problem 243](#):
$R_1 = 25(2.10)/2.70$
$R_1 = 19.44 \text{ kN}$

$R_2 = 50(0.90)/2.70$
$R_2 = 16.67 \text{ kN}$

$R_A = R_1 + R_2$
$R_A = 19.44 + 16.67$
$R_A = 36.11 \text{ kN}$

For segment BC
$P_{BC} + 25 = R_A$
$P_{BC} + 25 = 36.11$
$P_{BC} = 11.11 \text{ kN}$

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{11.11(1000)}{500}$$
$\sigma_{BC} = 22.22 \text{ MPa}$ → ***answer***

Problem 247

The composite bar in Fig. P-247 is stress-free before the axial loads P_1 and P_2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P_1 = 150 kN and P_2 = 90 kN.

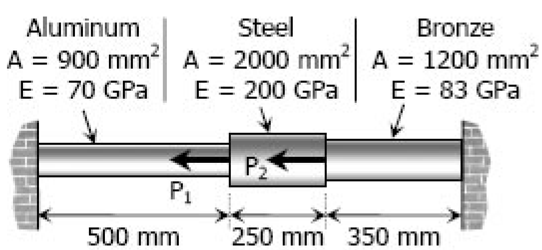
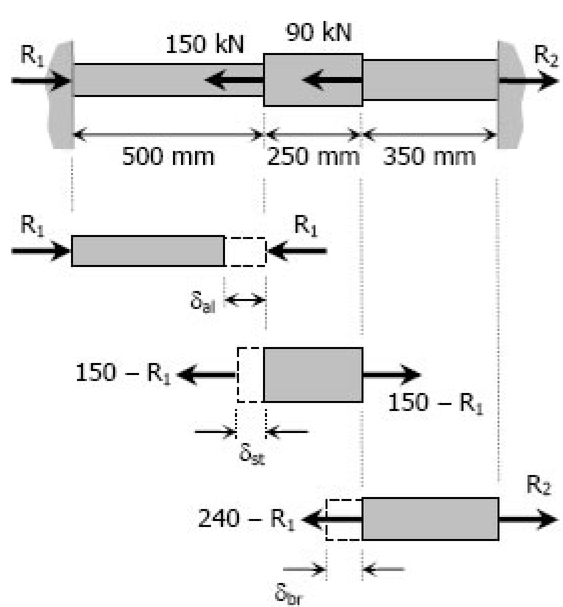


Figure P-247 and P-248

Solution 247



From the FBD of each material shown:
δ_{al} is shortening
δ_{st} and δ_{br} are lengthening

$R_2 = 240 - R_1$.
$P_{al} = R_1$
$P_{st} = 150 - R_1$
$P_{br} = R_2 = 240 - R_1$

$\delta_{al} = \delta_{st} + \delta_{br}$
$\left(\dfrac{PL}{AE}\right)_{al} = \left(\dfrac{PL}{AE}\right)_{st} + \left(\dfrac{PL}{AE}\right)_{br}$
$\dfrac{R_1\,(500)}{900(70\,000)} = \dfrac{(150 - R_1)(250)}{2000(200\,000)} + \dfrac{(240 - R_1)(350)}{1200(83\,000)}$
$\dfrac{R_1}{126\,000} = \dfrac{150 - R_1}{1\,600\,000} + \dfrac{(240 - R_1)7}{1\,992\,000}$
$\frac{1}{63}R_1 = \frac{1}{800}(150 - R_1) + \frac{7}{996}(240 - R_1)$
$(\frac{1}{63} + \frac{1}{800} + \frac{7}{996})R_1 = \frac{1}{800}(150) + \frac{7}{996}(240)$
$R_1 = 77.60\ \text{ kN}$

$P_{al} = R_1 = 77.60\,\text{kN}$
$P_{st} = 150 - 77.60 = 72.40\,\text{kN}$
$P_{br} = 240 - 77.60 = 162.40\,\text{kN}$

$\sigma = P/A$

$\sigma_{al} = 77.60(1000)/900$
$\sigma_{al} = 86.22\,\text{MPa}$ →***answer***

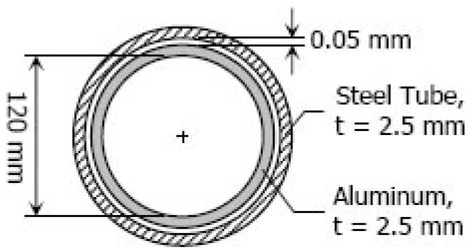
$\sigma_{st} = 72.40(1000)/2000$
$\sigma_{st} = 36.20\,\text{MPa}$ →***answer***

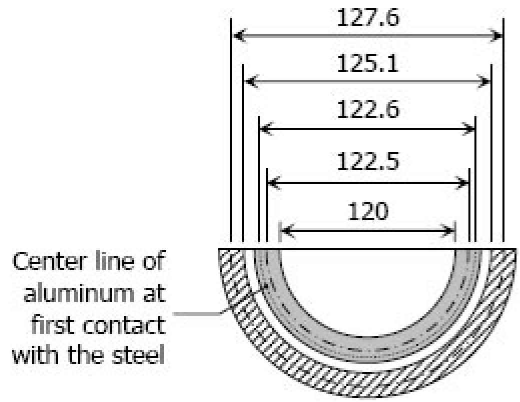
$\sigma_{br} = 162.40(1000)/1200$
$\sigma_{br} = 135.33\,\text{MPa}$ →***answer***

Problem 249

There is a radial clearance of 0.05 mm when a steel tube is placed over an aluminum tube. The inside diameter of the aluminum tube is 120 mm, and the wall thickness of each tube is 2.5 mm. Compute the contact pressure and tangential stress in each tube when the aluminum tube is subjected to an internal pressure of 5.0 MPa.

Solution 249





Internal pressure of aluminum tube to cause contact with the steel:

$$\delta_{al} = \left(\frac{\sigma\, L}{E}\right)_{al}$$

$$\pi(122.6 - 122.5) = \frac{\sigma_1\,(122.5\pi)}{70\,000}$$

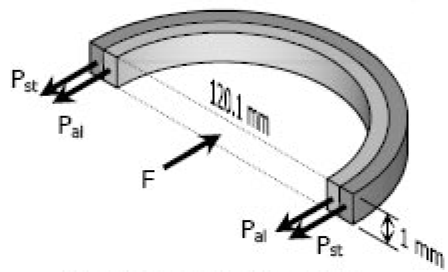
$$\sigma_1 = 57.143\ \text{MPa}$$

$$\frac{p_1\, D}{2t} = 57.143$$

$$\frac{p_1\,(120)}{2(2.5)} = 57.143$$

$p_1 = 2.381\ \text{MPa}$ $\rightarrow$pressure that causes aluminum to contact with the steel, further increase of pressure will expand both aluminum and steel tubes.

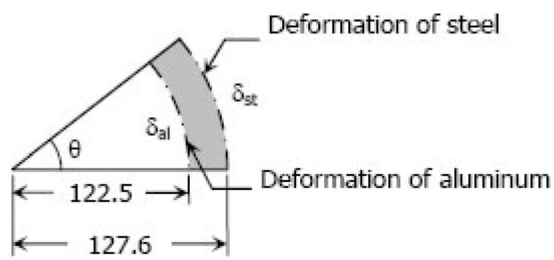
Let p_c = contact pressure between steel and aluminum tubes



FBD for p ≥ 2.381 MPa

$2P_{st} + 2P_{al} = F$
$2P_{st} + 2P_{al} = 5.0(120.1)(1)$
$P_{st} + P_{al} = 300.25$ →Equation (1)

The relationship of deformations is (from the figure):



Geometric relation of deformations

$\delta_{st} = 127.6\theta$
$\theta = \delta_{st}/127.6$

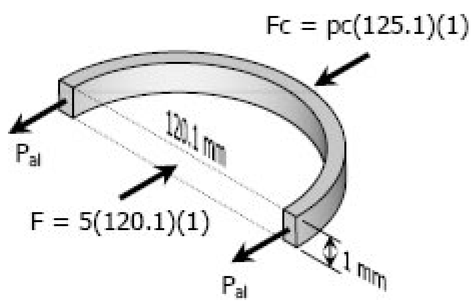
$\delta_{al} = 122.5\theta$
$\delta_{al} = 122.5(\delta_{st}/127.6)$
$\delta_{al} = 0.96\,\delta_{st}$
$\left(\dfrac{PL}{AE}\right)_{al} = 0.96\,\left(\dfrac{PL}{AE}\right)_{st}$
$\dfrac{P_{al}\,(122.5\pi)}{2.5(70\,000)} = 0.96\,\left[\dfrac{P_{st}\,(127.6)}{2.5(200\,000)}\right]$
$P_{al} = 0.35P_{st}$ →Equation (2)

From Equation (1)

$P_{st} + 0.35P_{st} = 300.25$
$P_{st} = 222.41\,\mathrm{N}$

$P_{al} = 0.35(222.41)$
$P_{al} = 77.84\,\mathrm{N}$

Contact Force



$F_c + 2P_{st} = F$
$p_c(125.1)(1) + 2(77.84) = 5(120.1)(1)$
$p_c = 3.56 \;\; \mathrm{MPa}$ →***answer***

Problem 254

As shown, a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially stress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.

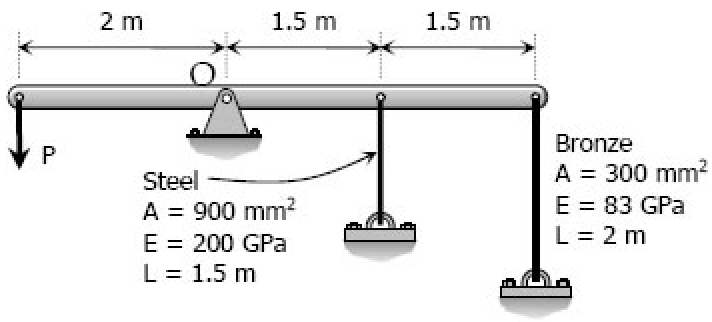
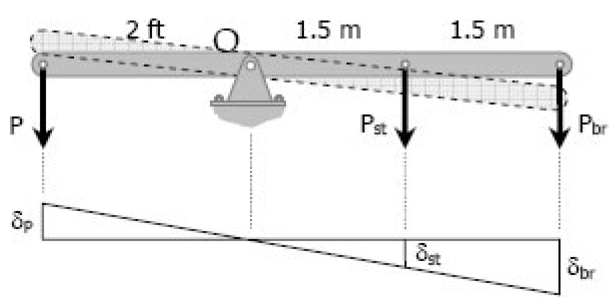


Figure P-254

Solution 254

$\Sigma M_O = 0$
$2P = 1.5P_{st} + 3P_{br}$
$2P = 1.5(\sigma_{st}\,A_{st}) + 3(\sigma_{br}\,A_{br})$
$2P = 1.5\,[\,\sigma_{st}\,(900)\,] + 3\,[\,\sigma_{br}\,(300)\,]$
$2P = 1350\sigma_{st} + 900\sigma_{br}$
$P = 675\sigma_{st} + 450\sigma_{br}$



$$\frac{\delta_{br}}{3} = \frac{\delta_{st}}{1.5}$$
$$\delta_{br} = 2\delta_{st}$$
$$\left(\frac{\sigma\,L}{E}\right)_{br} = 2\left(\frac{\sigma\,L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(2)}{83} = 2\left[\frac{\sigma_{st}(1.5)}{200}\right]$$
$$\sigma_{br} = 0.6225\sigma_{st}$$

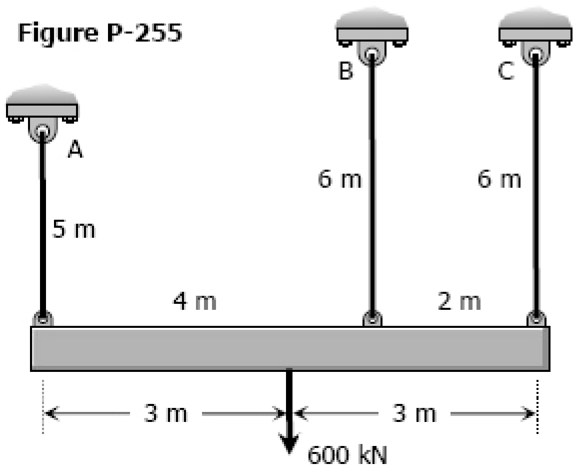
When $\sigma_{st} = 150\,\text{MPa}$
$$\sigma_{br} = 0.6225(150)$$
$\sigma_{br} = 93.375\,\text{MPa} > 70\,\text{MPa}$(*not ok!*)

When $\sigma_{br} = 70\,\text{MPa}$
$$70 = 0.6225\sigma_{st}$$
$\sigma_{st} = 112.45\,\text{MPa} < 150\,\text{MPa}$(*ok!*)

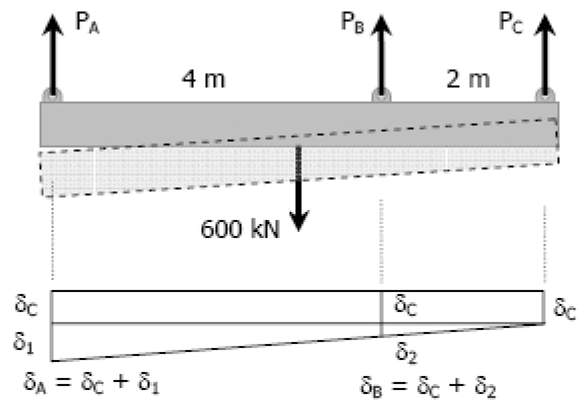
Use $\sigma_{st} = 112.45\,\text{MPa}$and $\sigma_{br} = 70\,\text{MPa}$
$$P = 675\sigma_{st} + 450\sigma_{br}$$
$$P = 675(112.45) + 450(70)$$
$$P = 107\,403.75\,\text{N}$$
$P = 107.4\,\text{kN}$ →***answer***

Problem 255

Shown in Fig. P-255 is a section through a balcony. The total uniform load of 600 kN is supported by three rods of the same area and material. Compute the load in each rod. Assume the floor to be rigid, but note that it does not necessarily remain horizontal.



Solution 255



$$\delta_B = \delta_C + \delta_2$$
$$\delta_2 = \delta_B - \delta_C$$
$$\frac{\delta_1}{6} = \frac{\delta_2}{2}$$
$$\delta_1 = 3\delta_2$$

$$\delta_A = \delta_C + \delta_1$$
$$\delta_A = \delta_C + 3\delta_2$$
$$\delta_A = \delta_C + 3(\delta_B - \delta_C)$$
$$\delta_A = 3\delta_B - 2\delta_C$$
$$\left(\frac{PL}{AE}\right)_A = 3\left(\frac{PL}{AE}\right)_B - 2\left(\frac{PL}{AE}\right)_C$$
$$\frac{P_A(5)}{AE} = \frac{3P_B(6)}{AE} - \frac{2P_C(6)}{AE}$$
$$P_A = 3.6P_B - 2.4P_C \rightarrow \text{Equation (1)}$$

$$\Sigma F_V = 0$$
$$P_A + P_B + P_C = 600$$
$$(3.6P_B - 2.4P_C) + P_B + P_C = 600$$
$$4.6P_B - 1.4P_C = 600 \rightarrow \text{Equation (2)}$$

$$\Sigma M_A = 0$$
$$4P_B + 6P_C = 3(600)$$
$$P_B = 450 - 1.5P_C$$ →Equation (3)

Substitute P_B = 450 - 1.5 P_C to Equation (2)

$$4.6(450 - 1.5P_C) - 1.4P_C = 600$$
$$8.3P_C = 1470$$
$$P_C = 177.11 \text{ kN}$$ →***answer***

From Equation (3)

$$P_B = 450 - 1.5(177.11)$$
$$P_B = 184.34 \text{ kN}$$ →***answer***

From Equation (1)

$$P_A = 3.6(184.34) - 2.4(177.11)$$
$$P_A = 238.56 \text{ kN}$$ →***answer***

Problem 256

Three rods, each of area 250 mm^2, jointly support a 7.5 kN load, as shown in Fig. P-256. Assuming that there was no slack or stress in the rods before the load was applied, find the stress in each rod. Use E_{st} = 200 GPa and E_{br} = 83 GPa.

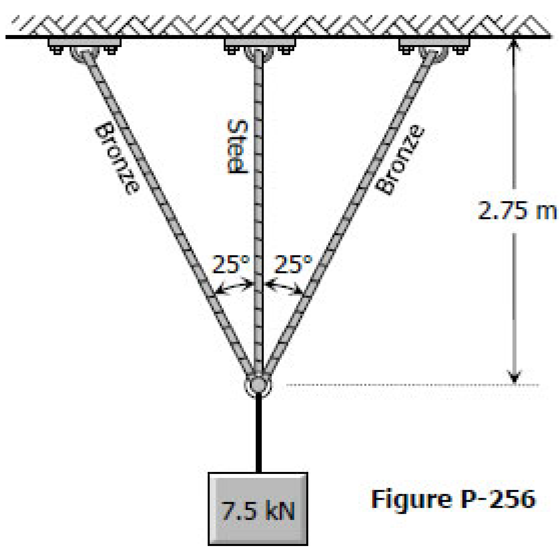
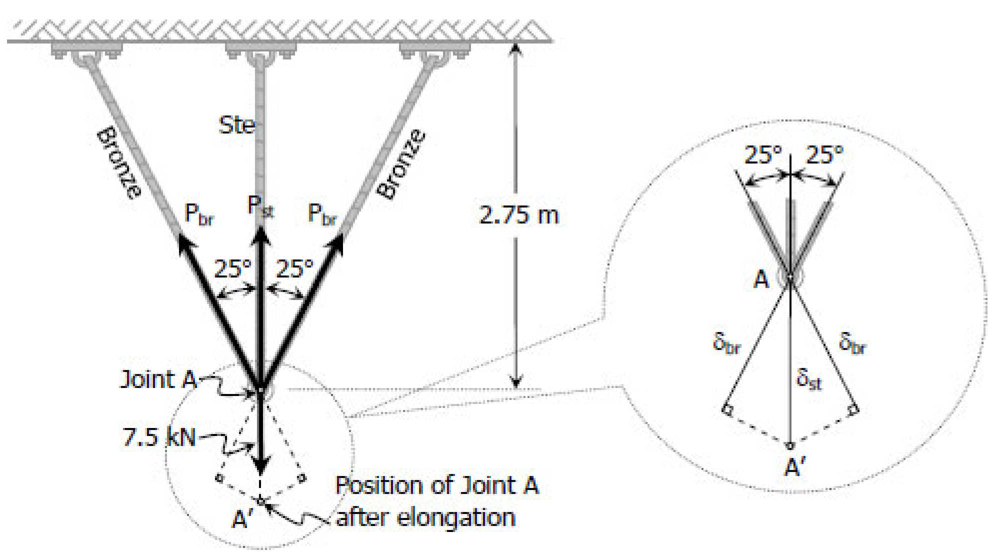


Figure P-256

Solution 256

$$\cos 25^\circ = \frac{2.75}{L_{br}}$$
$$L_{br} = 3.03 \text{ m}$$



$$\Sigma F_V = 0$$
$$2P_{br}\cos 25° + P_{st} = 7.5(1000)$$
$$P_{st} = 7500 - 1.8126P_{br}$$
$$\sigma_{st}\,A_{st} = 7500 - 1.8126\sigma_{br}\,A_{br}$$
$$\sigma_{st}(250) = 7500 - 1.8126\left[\,\sigma_{br}(250)\,\right]$$
$$\sigma_{st} = 30 - 1.8126\sigma_{br} \;\rightarrow$$ **Equation (1)**

$$\cos 25° = \frac{\delta_{br}}{\delta_{st}}$$
$$\delta_{br} = 0.9063\delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = 0.9063\left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(3.03)}{83} = 0.0963\left[\frac{\sigma_{st}(2.75)}{200}\right]$$
$$\sigma_{br} = 0.3414\sigma_{st} \;\rightarrow$$ **Equation (2)**

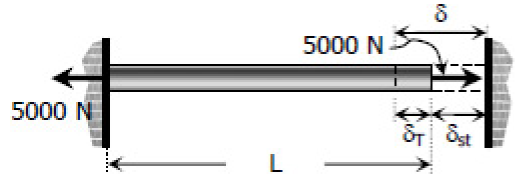
From Equation (1)
$$\sigma_{st} = 30 - 1.8126(0.3414\sigma_{st})$$
$$\sigma_{st} = 18.53 \;\text{ MPa} \;\rightarrow$$ ***answer***

From Equation (2)
$$\sigma_{br} = 0.3414(18.53)$$
$$\sigma_{br} = 6.33 \;\text{ MPa} \;\rightarrow$$ ***answer***

Problem 262

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

Solution 262



$$\delta = \delta_T + \delta_{st}$$

$$\frac{\sigma\, L}{E} = \alpha\, L\, (\Delta T) + \frac{PL}{AE}$$

$$\sigma = \alpha\, E\, (\Delta T) + \frac{P}{E}$$

$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$

$$A = \frac{5000}{36.4} 137.36$$

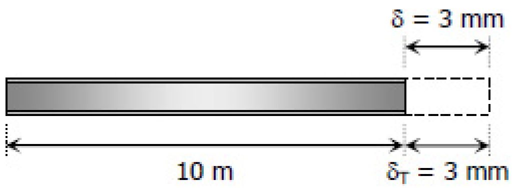
$\frac{1}{4}\pi d^2 = 137.36$

d$= 13.22 \text{ mm}$ $\to$ ***answer***

Problem 263

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

Solution 263



Temperature at which $\delta_T = 3\,\mathrm{mm}$:

$\delta_T = \alpha\, L\, (\Delta T)$
$\delta_T = \alpha\, L\, (T_f\, -\, T_i)$
$3 = (11.7 \times 10^{-6})(10\,000)(T_f\, -\, 15)$
$T_f = 40.64^\circ\mathrm{C}$ → ***answer***

Required stress:

$\delta = \delta_T$

$\dfrac{\sigma\, L}{E} = \alpha\, L\, (\Delta T)$

$\sigma = \alpha\, E\, (T_f\, -\, T_i)$
$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$
$\sigma = 60\ \ \mathrm{MPa}$ → ***answer***

Problem 265

A bronze bar 3 m long with a cross sectional area of 320 mm^2 is placed between two rigid walls as shown in Fig. P-265. At a temperature of -20°C, the gap Δ = 2.5 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use $\alpha = 18.0 \times 10^{-6}$ m/(m·°C) and E = 80 GPa.

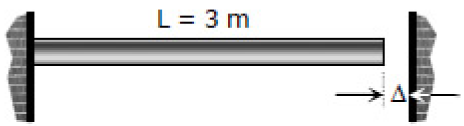
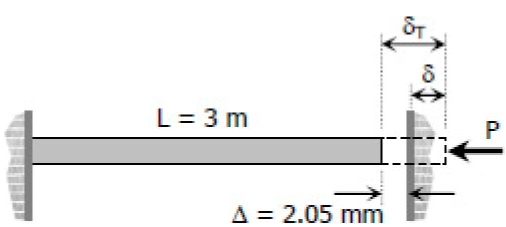


Figure P-265

Problem 265

$$\delta_T = \delta + \Delta$$

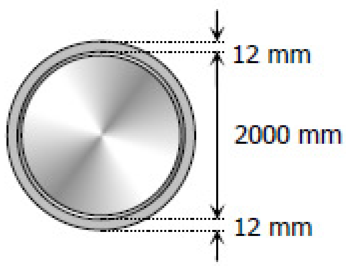


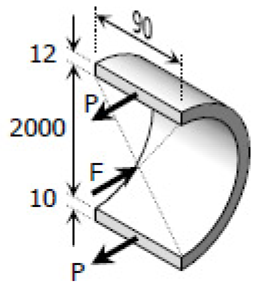
$$\alpha L \, (\Delta T) = \frac{\sigma L}{E} + 2.5$$
$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$
$$\Delta T = 70.6°\text{C}$$
$$T = 70.6 - 20$$
$T = 50.6°\text{C}$ → ***answer***

Problem 267

At a temperature of 80°C, a steel tire 12 mm thick and 90 mm wide that is to be shrunk onto a locomotive driving wheel 2 m in diameter just fits over the wheel, which is at a temperature of 25°C. Determine the contact pressure between the tire and wheel after the assembly cools to 25°C. Neglect the deformation of the wheel caused by the pressure of the tire. Assume α = 11.7 Î¼m/(m·°C) and E = 200 GPa.

Solution 267





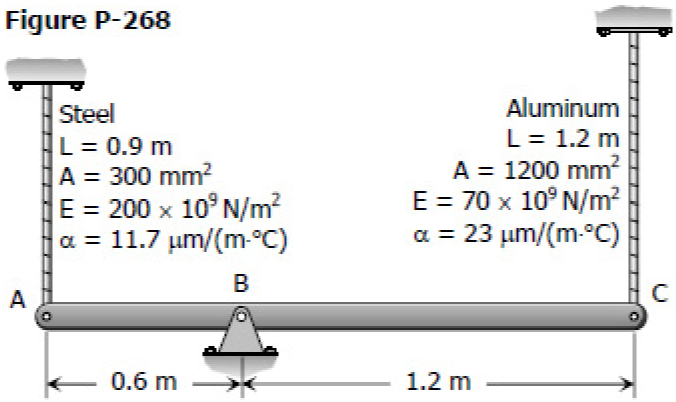
$$\delta = \delta_T$$
$$\frac{PL}{AE} = \alpha\, L\, \Delta T$$
$$P = \alpha\, \Delta T\, AE$$
$$P = (11.7 \times 10^{-6})(80 - 25)(90 \times 12)(200\,000)$$
$$P = 138\,996\,\text{N}$$

$$F = 2P$$
$$pDL = 2P$$
$$p(2000)(90) = 2(138\,996)$$
$$p = 1.5444\,\text{MPa}$$ →***answer***

Problem 268

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.

Figure P-268



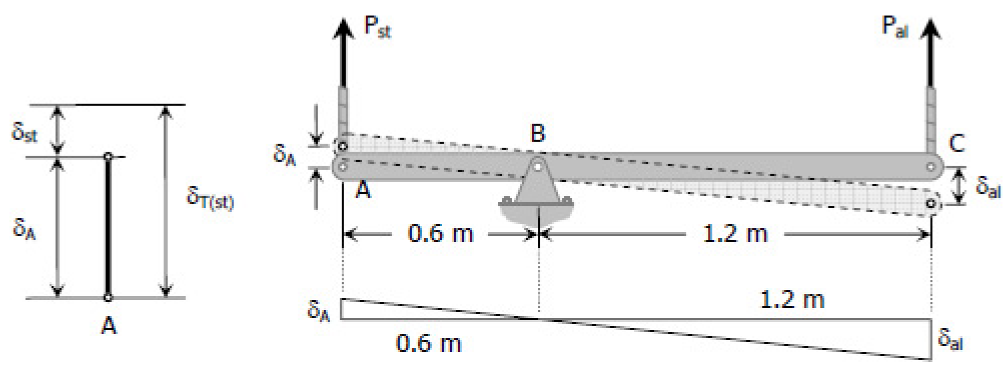
Solution 268

Contraction of steel rod, assuming complete freedom:

$$\delta_{T(st)} = \alpha \, L \, \Delta T$$
$$\delta_{T(st)} = (11.7 \times 10^{-6})(900)(40)$$
$$\delta_{T(st)} = 0.4212 \, \text{mm}$$

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as δ_A), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\frac{\delta_A}{0.6} = \delta_{al} 1.2$$
$$\delta_A = 0.5 \delta_{al}$$



$$\delta_{T(st)} - \delta_{st} = 0.5\delta_{al}$$

$$0.4212 - \left(\frac{PL}{AE}\right)_{st} = 0.5\left(\frac{PL}{AE}\right)_{al}$$

$$0.4212 - \frac{P_{st}(900)}{300(200\,000)} = 0.5\left[\frac{P_{al}(1200)}{1\,200(70\,000)}\right]$$

$28080 - P_{st} = 0.4762P_{al}$ →**Equation (1)**

$\Sigma M_B = 0$
$0.6P_{st} = 1.2P_{al}$
$P_{st} = 2P_{al}$ →**Equation (2)**

Equations (1) and (2)
$28\,080 - 2P_{al} = 0.4762P_{al}$
$P_{al} = 11\,340\,\text{N}$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11\,340}{1200}$$
$\sigma_{al} = 9.45$ MPa →***answer***

Problem 269

As shown in Fig. P-269, there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, Δ = 0.18 mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar, A = 500 mm^2, E = 120 GPa, and α = 16.8 μm/(m·°C). For the aluminum bar, A = 400 mm^2, E = 70 GPa, and α = 23.1 μm/(m·°C).

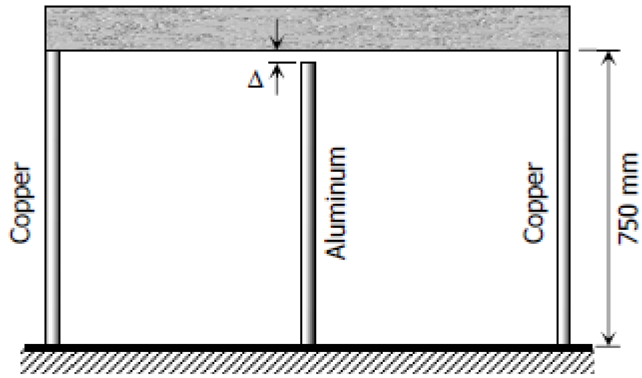
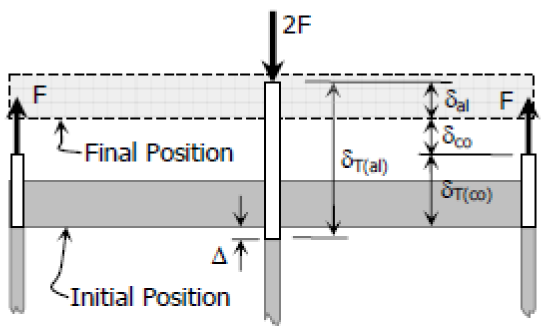


Figure P-269

Solution 269

Assuming complete freedom:

$$\delta_T = \alpha\, L\, \Delta T$$
$$\delta_{T(co)} = (16.8 \times 10^{-6})(750)(95 - 10)$$
$$\delta_{T(co)} = 1.071\ \text{mm}$$
$$\delta_{T(al)} = (23.1 \times 10^{-6})(750 - 0.18)(95 - 10)$$
$$\delta_{T(al)} = 1.472\ \text{mm}$$



From the figure:

$$\delta_{T(al)} - \delta_{al} = \delta_{T(co)} + \delta_{co}$$
$$1.472 - \left(\frac{PL}{AE}\right)_{al} = 1.071 + \left(\frac{PL}{AE}\right)_{co}$$
$$1.472 - \frac{2F(750 - 0.18)}{400(70\,000)} = 1.071 + \frac{F(750)}{500(120\,000)}$$
$$0.401 = (6.606 \times 10^{-5})\, F$$
$$F = 6070.37\ \text{N}$$

$$P_{co} = F = 6070.37\ \text{N}$$
$$P_{al} = 2F = 12\,140.74\ \text{N}$$

$$\sigma = P/A$$
$$\sigma_{co} = \frac{6070.37}{500} = 12.14\ \text{MPa} \rightarrow$$ ***answer***
$$\sigma_{al} = \frac{12\,140.74}{400} = 30.35\ \text{MPa} \rightarrow$$ ***answer***

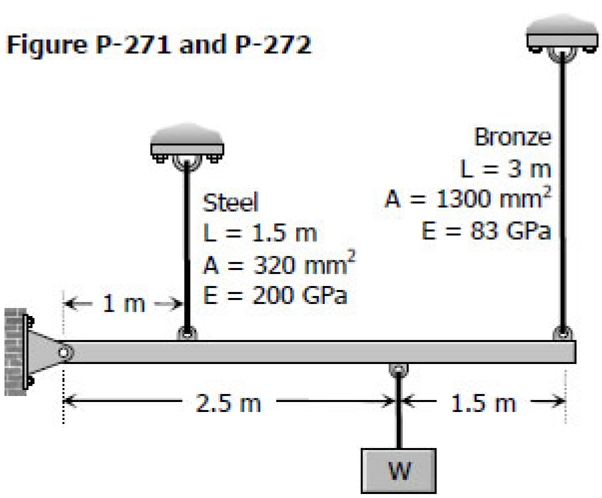
Problem 272

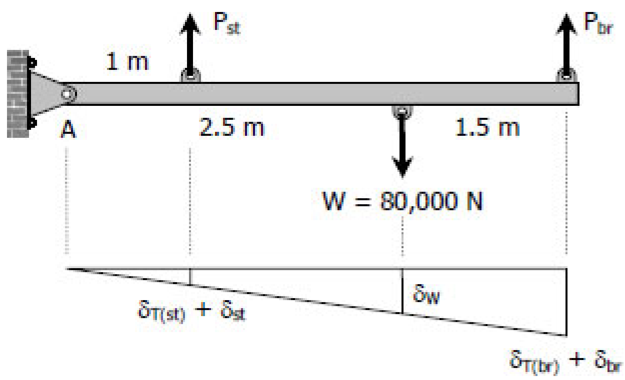
For the assembly in Fig. 271, find the stress in each rod if the temperature rises 30°C after a load W = 120 kN is applied.

Solution 272

$$\Sigma M_A = 0$$
$$4P_{br} + P_{st} = 2.5(80\,000)$$
$$4\sigma_{br}(1300) + \sigma_{st}(320) = 2.5(80\,000)$$
$$16.25\sigma_{br} + \sigma_{st} = 625$$
$$\sigma_{st} = 625 - 16.25\sigma_{br} \rightarrow$$ ***Equation (1)***

Figure P-271 and P-272



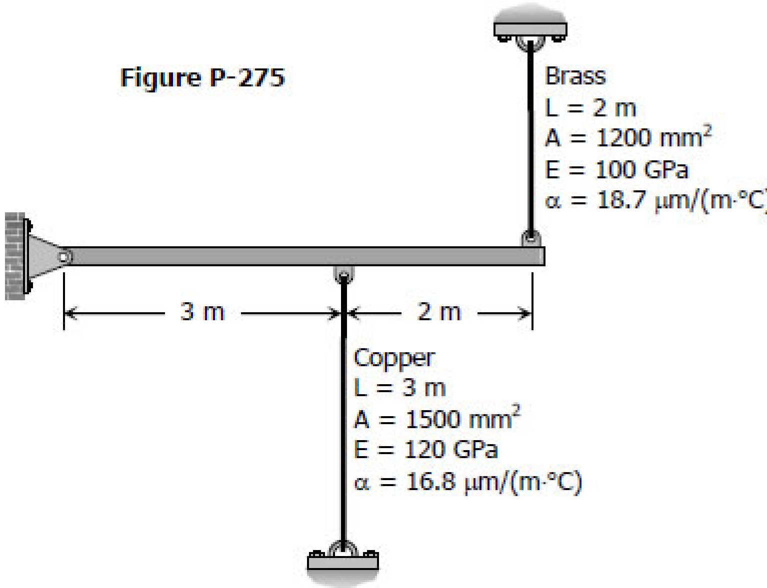


$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$
$$\delta_{T(st)} + \delta_{st} = 0.25\,[\delta_{T(br)} + \delta_{br}]$$
$$(\alpha\,L\,\Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha\,L\,\Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$
$$(11.7{\times}10^{-6})(1500)(30) + \frac{\sigma_{st}(1500)}{200\,000} = 0.25\left[(18.9\times10^{-6})(3000)(30) + \frac{\sigma_{br}(3000)}{83\,000}\right]$$
$$0.5265 + 0.0075\sigma_{st} = 0.425\,25 + 0.009\,04\sigma_{br}$$
$$0.0075\sigma_{st} - 0.009\,04\sigma_{br} = -0.101\,25$$
$$0.0075\,(625 - 16.25\sigma_{br}) - 0.009\,04\sigma_{br} = -0.101\,25$$
$$4.6875 - 0.121\,875\sigma_{br} - 0.009\,04\sigma_{br} = -0.101\,25$$
$$4.788\,75 = 0.130\,915\sigma_{br}$$
$$\sigma_{br} = 36.58 deg\,; \mathrm{C}$$ ***answer***

$$\sigma_{st} = 625 - 16.25(36.58)$$
$$\sigma_{st} = 30.58\ \mathrm{deg;\ C}$$ ***answer***

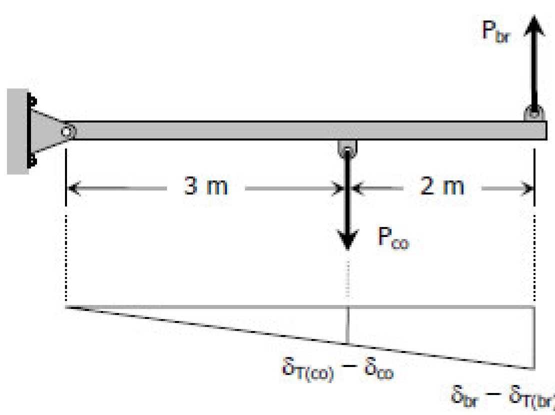
Problem 275

A rigid horizontal bar of negligible mass is connected to two rods as shown in [Fig. P-275.]() If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.



Solution 275

$$\Sigma M_{hinge\ support} = 0$$
$$5P_{br} - 3P_{co} = 0$$
$$5\sigma_{br}A_{br} - 3\sigma_{co}A_{co} = 0$$
$$5(90)(1200) - 3\sigma_{co}(1500) = 0$$
$$\sigma_{co} = 120\,\text{MPa}$$



$$\delta = \frac{\sigma L}{E}$$
$$\delta_{br} = \frac{90(2000)}{100\,000} = 1.8\text{ mm}$$
$$\delta_{co} = \frac{120(3000)}{120\,000} = 3\text{ mm}$$

$$\frac{\delta_{T(co)} - \delta_{co}}{3} = \frac{\delta_{br} - \delta_{T(br)}}{5}$$
$$5\delta_{T(co)} - 5\delta_{co} = 3\delta_{br} - 3\delta_{T(br)}$$
$$5(16.8 \times 10^{-6})(3000)\,\Delta T - 5(3) = 3(1.8) - 3(18.7 \times 10^{-6})(2000)\,\Delta T$$
$$0.3642\,\Delta T = 20.4$$
$\Delta T = 56.01°\text{C}$ drop in temperature ***answer***

Problem 276

Four steel bars jointly support a mass of 15 Mg as shown in Fig. P-276. Each bar has a cross-sectional area of 600 mm^2. Find the load carried by each bar after a temperature rise of 50°C. Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

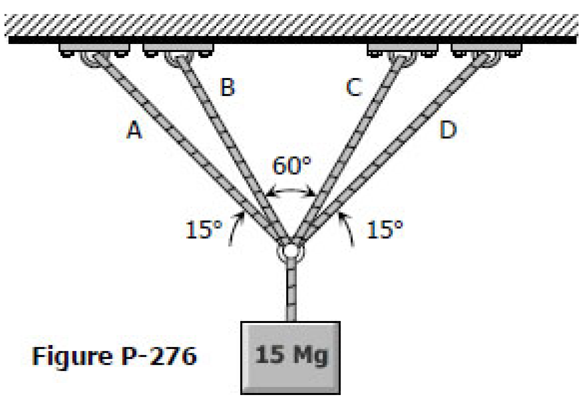
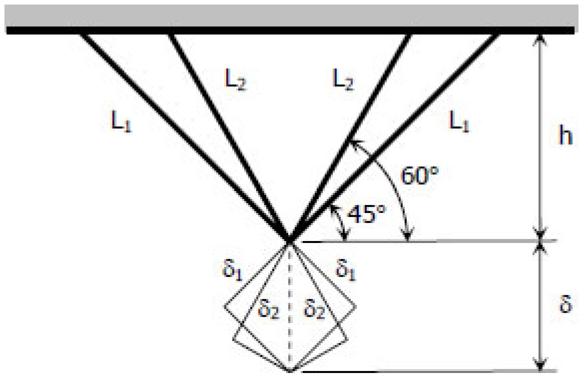


Figure P-276

Solution 276

$$h = L_1 \sin 45^\circ$$
$$h = L_2 \sin 60^\circ$$



$$h = h$$
$$L_1 \sin 45^\circ = L_2 \sin 60^\circ$$
$$L_1 = 1.2247 L_2$$
$$\delta_1 = \delta \sin 45^\circ$$
$$\delta_2 = \delta \sin 60^\circ$$

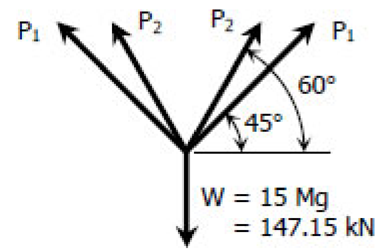
$$\frac{\delta_1}{\delta_2} = \frac{\delta \sin 45^\circ}{\delta \sin 60^\circ}$$
$$\delta_1 = 0.8165 \delta_2$$

$$\alpha L_1 \Delta T + \frac{P_1 L_1}{AE} = 0.8165 \left[\alpha L_2 \Delta T + \frac{P_2 L_2}{AE} \right]$$

$$(11.7 \times 10^{-6}) L_1 (50) + \frac{P_1 L_1}{600(200\,000)} = 0.8165 \left[(11.7 \times 10^{-6})(50) + \frac{P_2 L_2}{600(200\,000)} \right]$$

$$70,200 L_1 + P_1 L_1 = 0.8165(70,200 L_2 + P_2 L_2)$$
$$(70,200 + P_1) L_1 = 0.8165(70,200 + P_2) L_2$$
$$(70,200 + P_1) 1.2247 L_2 = 0.8165(70,200 + P_2) L_2$$
$$1.5(70,200 + P_1) = 70,200 + P_2$$
$$P_2 = 1.5 P_1 + 35,100$$ → ***Equation (1)***

$$\Sigma F_V = 0$$
$$2(P_1 \sin 45°) + 2(P_2 \sin 60°) = 147.15(1000)$$
$$P_1 \sin 45° + P_2 \sin 60° = 72,575$$
$$P_1 \sin 45° + (1.5P_1 + 35,100) \sin 60° = 72,575$$
$$0.7071P_1 + 1.299P_1 + 30,397.49 = 72,575$$
$$2.0061P_1 = 42,177.51$$
$$P_1 = 21,024.63 \text{ N}$$



$$P_2 = 1.5(21,024.63) + 35,100$$
$$P_2 = 66,636.94 \text{ N}$$

$P_A = P_D = P_1 = 21.02$ kN ***answer***
$P_B = P_C = P_2 = 66.64$ kN ***answer***

Problem 304

A steel shaft 3 ft long that has a diameter of 4 in is subjected to a torque of 15 kip·ft. Determine the maximum shearing stress and the angle of twist. Use G = 12 × 10^6 psi.

Solution 304

$$\tau_{max} = \frac{16T}{\pi D^3} = \frac{16(15)(1000)(12)}{\pi(4^3)}$$

$$\tau_{max} = 14324 \, \text{psi}$$

$\tau_{max} = 14.3 \, \text{ksi}$***answer***

$$\theta = \frac{TL}{JG} = \frac{15(3)(1000)(12^2)}{\frac{1}{32}\pi(4^4)(12 \times 10^6)}$$

$$\theta = 0.0215 \, \text{rad}$$

$\theta = 1.23°$ ***answer***

Problem 305

What is the minimum diameter of a solid steel shaft that will not twist through more than 3° in a 6-m length when subjected to a torque of 12 kN·m? What maximum shearing stress is developed? Use G = 83 GPa.

Solution 305

$$\theta = \frac{TL}{JG}$$
$$3°\left(\frac{\pi}{180°}\right) = \frac{12(6)(1000^3)}{\frac{1}{32}\pi d^4(83\,000)}$$
$d = 113.98 \, \text{mm}$***answer***

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(12)(1000^2)}{\pi(113.98^3)}$$
$$\tau_{max} = 41.27\text{ MPa}$$ ***answer***

Problem 306

A steel marine propeller shaft 14 in. in diameter and 18 ft long is used to transmit 5000 hp at 189 rpm. If G = 12 × 10^6 psi, determine the maximum shearing stress.

Solution 306

$$T = \frac{P}{2\pi f} = \frac{5000(396\,000)}{2\pi(189)}$$
$$T = 1\,667\,337.5\text{ lb}\cdot\text{in}$$

$$\tau_{max} = \frac{16T}{\pi d^3} = \frac{16(1\,667\,337.5)}{\pi(14^3)}$$
$$\tau_{max} = 3094.6\text{ psi}$$ ***answer***

Problem 307

A solid steel shaft 5 m long is stressed at 80 MPa when twisted through 4°. Using G = 83 GPa, compute the shaft diameter. What power can be transmitted by the shaft at 20 Hz?

Solution 307

$$\theta = \frac{TL}{JG}$$
$$4^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{T(5)(1000)}{\frac{1}{32}\pi d^4(83\,000)}$$
$$T = 0.1138d^4$$

$$\tau_{max} = \frac{16T}{\pi d^3}$$
$$80 = \frac{16(0.1138^4)}{\pi d^3}$$
$$d = 138\text{ mm}$$ ***answer***

$$T = \frac{P}{2\pi f}$$
$$0.1138d^4 = \frac{P}{2\pi(20)}$$
$$P = 14.3d^4 = 14.3(1384)$$
$$P = 5\,186\,237\,285\text{ N}\cdot\text{mm/sec}$$
$$P = 5\,186\,237.28\text{ W}$$
$$P = 5.19\text{ MW}$$ ***answer***

Problem 308

A 2-in-diameter steel shaft rotates at 240 rpm. If the shearing stress is limited to 12 ksi, determine the maximum horsepower that can be transmitted.

Solution 308

$$\tau_{max} = \frac{16T}{\pi d^3}$$
$$12(1000) = \frac{16T}{\pi(2^3)}$$
$$T = 18\,849.56\text{ lb}\cdot\text{in}$$

$$T = \frac{P}{2\pi f}$$
$$18\,849.56 = \frac{P(396\,000}{2\pi(240)}$$
$P = 71.78\ \text{hp}$***answer***

Problem 309

A steel propeller shaft is to transmit 4.5 MW at 3 Hz without exceeding a shearing stress of 50 MPa or twisting through more than 1° in a length of 26 diameters. Compute the proper diameter if G = 83 GPa.

Solution 309

$$T = \frac{P}{2\pi f} = \frac{4.5(1\,000\,000)}{2\pi(3)}$$
$$T = 238\,732.41\ \text{N} \cdot \text{m}$$

Based on maximum allowable shearing stress:

$$\tau_{max} = \frac{16T}{\pi d^3}$$
$$50 = \frac{16(238\,732.41)(1000)}{\pi d^3}$$
$$d = 289.71\ \text{mm}$$

Based on maximum allowable angle of twist:

$$\theta = \frac{TL}{JG}$$
$$1^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{238\,732.41(26d)(1000)}{\frac{1}{32}\pi d^4(83\,000)}$$
$$d = 352.08\ \text{mm}$$

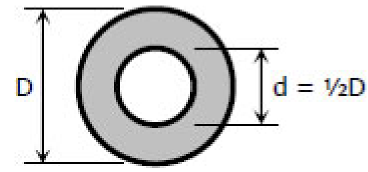
Use the bigger diameter, **d = 352 mm *answer***

Problem 310

Show that the hollow circular shaft whose inner diameter is half the outer diameter has a torsional strength equal to 15/16 of that of a solid shaft of the same outside diameter.

Solution 310

Hollow circular shaft:



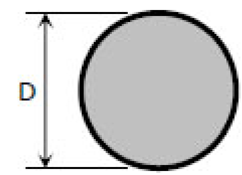
$$\tau_{max-hollow} = \frac{16TD}{\pi(D^4 - d^4)}$$

$$\tau_{max-hollow} = \frac{16TD}{\pi\left[\,D^4 - (\frac{1}{2}D)^4\,\right]}$$

$$\tau_{max-hollow} = \frac{16TD}{\pi(\frac{15}{16}D^4)}$$

$$\tau_{max-hollow} = \frac{16^2T}{15\pi D^3}$$

Solid circular shaft:



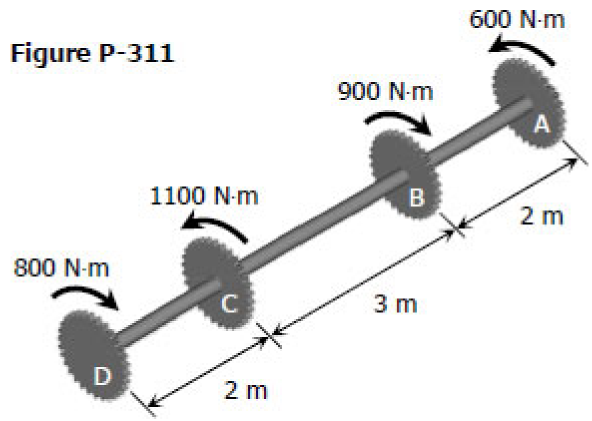
$$\tau_{max-solid} = \frac{16T}{\pi D^3}$$

$$\tau_{max-solid} = \frac{15}{16}\left[\frac{16^2T}{15\pi D^3}\right]$$

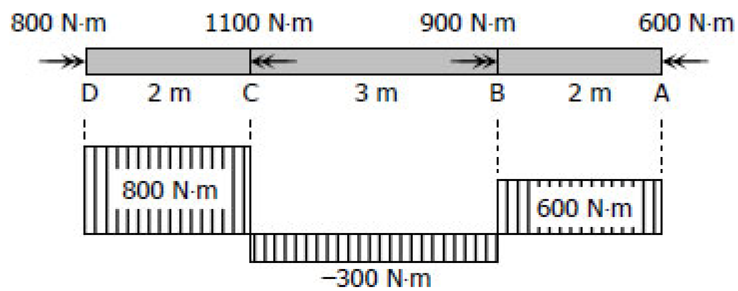
$$\tau_{max-solid} = \tfrac{15}{16} \times \tau_{max-hollow}$$ ***ok!***

Problem 311

An aluminum shaft with a constant diameter of 50 mm is loaded by torques applied to gears attached to it as shown in Fig. P-311. Using G = 28 GPa, determine the relative angle of twist of gear D relative to gear A.



Problem 311



$$\theta = \frac{TL}{JG}$$

Rotation of D relative to A:

$$\theta_{D/A} = \frac{1}{JG}\Sigma TL$$

$$\theta_{D/A} = \frac{1}{\frac{1}{32}\pi(50^4)(28\,000)}\left[\,800(2) - 300(3) + 600(2)\,\right](1000^2)$$

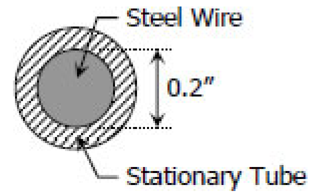
$$\theta_{D/A} = 0.1106\,\mathrm{rad}$$

$$\theta_{D/A} = 6.34^\circ$$ ***answer***

Problem 312

A flexible shaft consists of a 0.20-in-diameter steel wire encased in a stationary tube that fits closely enough to impose a frictional torque of 0.50 lb·in/in. Determine the maximum length of the shaft if the shearing stress is not to exceed 20 ksi. What will be the angular deformation of one end relative to the other end? G = 12 × 10^6 psi.

Solution 312



$$\tau_{max} = \frac{16T}{\pi d^3}$$
$$20(1000) = \frac{16T}{\pi (0.20)^3}$$
$$T = 10\pi \text{ lb} \cdot \text{in}$$

$$L = \frac{T}{0.50 \text{ lb} \cdot \text{in/in}}$$
$$L = \frac{10\pi \text{ lb} \cdot \text{in}}{0.50 \text{ lb} \cdot \text{in/in}}$$
$$L = 20\pi \text{ in} = 62.83 \text{ in}$$

$$\theta = \frac{TL}{JG}$$

If θ = dθ, T = 0.5L and L = dL

$$\int d\theta = \frac{1}{JG} \int_0^{20\pi} (0.5L)\, dL$$
$$\theta = \left[\frac{0.5L^2}{2} \right]_0^{2\pi} = \frac{1}{JG} \left[0.25(20\pi)^2 - 0.25(0)^2 \right]$$
$$\theta = \frac{100\pi^2}{\frac{1}{32}\pi(0.20^4)(12 \times 10^6)}$$
$\theta = 0.5234 \text{ rad} = 30°$ ***answer***

Problem 314

The steel shaft shown in Fig. P-314 rotates at 4 Hz with 35 kW taken off at A, 20 kW removed at B, and 55 kW applied at C. Using G = 83 GPa, find the maximum shearing stress and the angle of rotation of gear A relative to gear C.

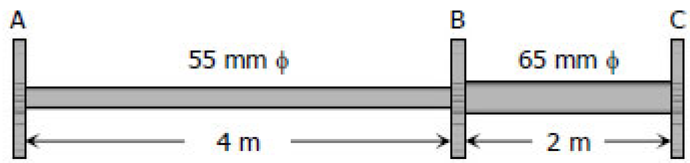


Figure P-314

Solution 314

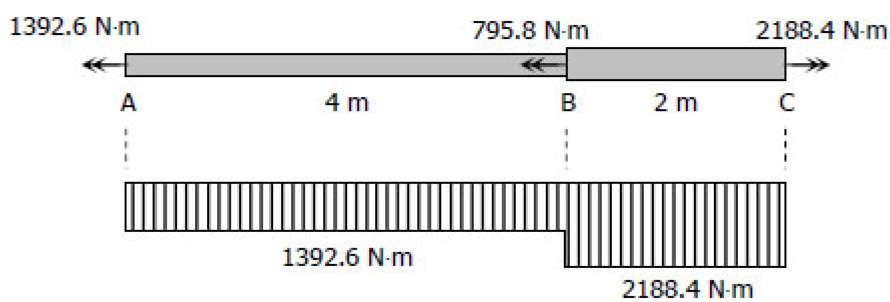
$$T = \frac{P}{2\pi f}$$

$$T_A = \frac{-35(1000)}{2\pi(4)} = -1392.6\ \mathrm{N \cdot m}$$

$$T_B = \frac{-20(1000)}{2\pi(4)} = -795.8\ \mathrm{N \cdot m}$$

$$T_B = \frac{55(1000)}{2\pi(4)} = 2188.4\ \mathrm{N \cdot m}$$

Relative to C:



$$\tau_{max} = \frac{16T}{\pi d^3}$$
$$\tau_{AB} = \frac{16(1392.6)(1000)}{\pi(55^3)} = 42.63\,\text{MPa}$$
$$\tau_{BC} = \frac{16(2188.4)(1000)}{\pi(65^3)} = 40.58\,\text{MPa}$$
$$\therefore\ \tau_{max} = \tau_{AB} = 42.63\,\text{MPa}$$ ***answer***

$$\theta = \frac{TL}{JG}$$
$$\theta_{A/C} = \frac{1}{G}\Sigma\frac{TL}{J}$$
$$\theta_{A/C} = \frac{1}{83\,000}\left[\frac{1392.6(4)}{\frac{1}{32}\pi(55^4)} + \frac{2188.4(2)}{\frac{1}{32}\pi(65^4)}\right](1000^2)$$
$$\theta_{A/C} = 0.104\,796\,585\,\text{rad}$$
$$\theta_{A/C} = 6.004^\circ$$ ***answer***

Problem 315

A 5-m steel shaft rotating at 2 Hz has 70 kW applied at a gear that is 2 m from the left end where 20 kW are removed. At the right end, 30 kW are removed and another 20 kW leaves the shaft at 1.5 m from the right end. (a) Find the uniform shaft diameter so that the shearing stress will not exceed 60 MPa. (b) If a uniform shaft diameter of 100 mm is specified, determine the angle by which one end of the shaft lags behind the other end. Use G = 83 GPa.

Solution 315

$$T = \frac{P}{2\pi f}$$
$$T_A = T_C = \frac{-20(1000)}{2\pi(2)} = -1591.55\,\text{N}\cdot\text{m}$$
$$T_B = \frac{70(1000)}{2\pi(2)} = 5570.42\,\text{N}\cdot\text{m}$$
$$T_D = \frac{-30(1000)}{2\pi(2)} = -2387.32\,\text{N}\cdot\text{m}$$

Problem 316

A compound shaft consisting of a steel segment and an aluminum segment is acted upon by two torques as shown in Fig. P-316. Determine the maximum permissible value of T subject to the following conditions: $\tau_{st} \leq 83$ MPa, $\tau_{al} \leq 55$ MPa, and the angle of rotation of the free end is limited to 6°. For steel, G = 83 GPa and for aluminum, G = 28 GPa.

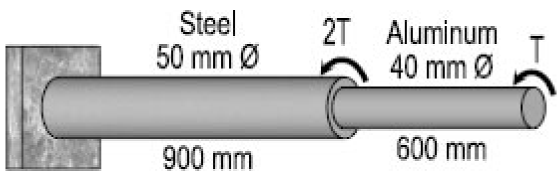
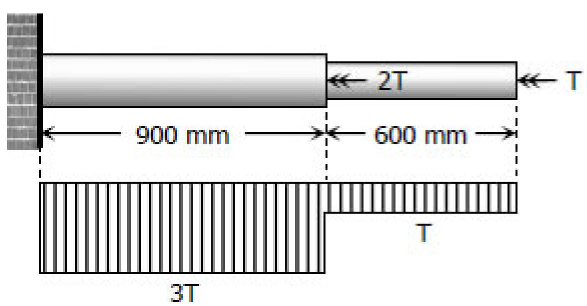


Figure P-316

Solution 316



Based on maximum shearing stress $\tau_{max} = 16T / \pi d^3$:

Steel

$$\tau_{st} = \frac{16(3T)}{\pi(50^3)} = 83$$

$$T = 679\,042.16 \text{ N} \cdot \text{mm}$$

$$T = 679.04 \text{ N} \cdot \text{m}$$

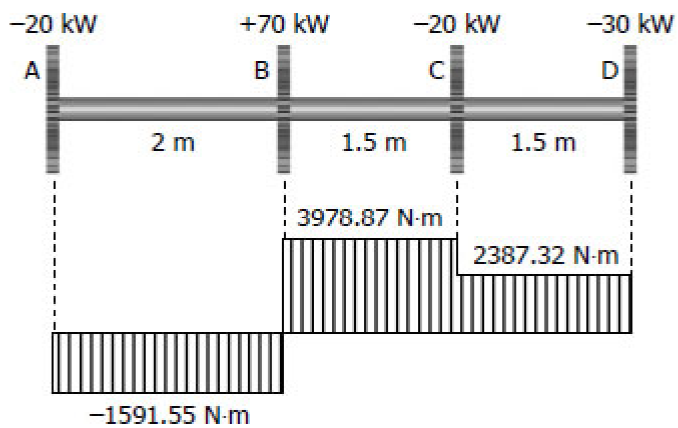
Aluminum

$$\tau_{al} = \frac{16T}{\pi(40^3)} = 55$$
$$T = 691\,150.38\ \mathrm{N\cdot mm}$$
$$T = 691.15\ \mathrm{N\cdot m}$$

Based on maximum angle of twist:

$$\theta = \left(\frac{TL}{JG}\right)_{st} + \left(\frac{TL}{JG}\right)_{al}$$
$$6^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{3T(900)}{\frac{1}{32}\pi(50^4)(83\,000)} + \frac{T(600)}{\frac{1}{32}\pi(40^4)(28\,000)}$$
$$T = 757\,316.32\ \mathrm{N\cdot mm}$$
$$T = 757.32\ \mathrm{N\cdot m}$$

Use T = 679.04 N·m ***answer***



Part (a)

$$\tau_{max} = \frac{16T}{\pi d^3}$$

For AB $$60 = \frac{16(1591.55)(1000)}{\pi d^3}$$

$$d = 51.3 \text{ mm}$$

For BC $$60 = \frac{16(3978.87)(1000)}{\pi d^3}$$

$$d = 69.6 \text{ mm}$$

For CD $$60 = \frac{16(2387.32)(1000)}{\pi d^3}$$

$$d = 58.7 \text{ mm}$$

Use d = 69.6 mm ***answer***

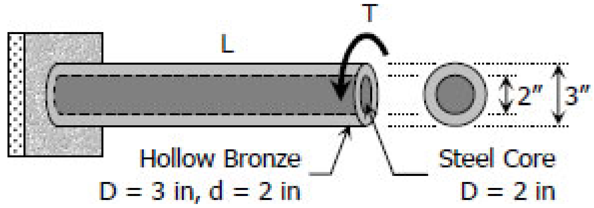
Part (b)

$$\theta = \frac{TL}{JG}$$
$$\theta_{D/A} = \frac{1}{JG}\Sigma TL$$
$$\theta_{D/A} = \frac{1}{\frac{1}{32}\pi(100^4)(83\,000)}\left[-1591.55(2)+3978.87(1.5)+2387.32(1.5)\right](1000^2)$$
$$\theta_{D/A} = 0.007\,813 \text{ rad}$$
$$\theta_{D/A} = 0.448^\circ$$ ***answer***

Problem 317

A hollow bronze shaft of 3 in. outer diameter and 2 in. inner diameter is slipped over a solid steel shaft 2 in. in diameter and of the same length as the hollow shaft. The two shafts are then fastened rigidly together at their ends. For bronze, $G = 6 \times 10^6$ psi, and for steel, $G = 12 \times 10^6$ psi. What torque can be applied to the composite shaft without exceeding a shearing stress of 8000 psi in the bronze or 12 ksi in the steel?

Solution 317



$$\theta_{st} = \theta_{br}$$
$$\left(\frac{TL}{JG}\right)_{st} = \left(\frac{TL}{JG}\right)_{br}$$
$$\frac{T_{st}L}{\frac{1}{32}\pi(2^4)(12\times10^6)} = \frac{T_{br}L}{\frac{1}{32}\pi(3^4-2^4)(6\times10^6)}$$
$$\frac{T_{st}}{192\times10^6} = \frac{T_{br}}{390\times10^6} \rightarrow$$ **Equation (1)**

Applied Torque = Resisting Torque
$$T = T_{st} + T_{br} \rightarrow$$ **Equation (2)**

Equation (1) with T_{st} in terms of T_{br} and Equation (2)
$$T = \frac{192\times10^6}{390\times10^6}T_{br} + T_{br}$$
$$T_{br} = 0.6701T$$

Equation (1) with T_{br} in terms of T_{st} and Equation (2)
$$T = T_{st} + \frac{390\times10^6}{192\times10^6}T_{br}$$
$$T_{st} = 0.3299T$$

Based on hollow bronze (T_{br} = 0.6701T)
$$\tau_{max} = \left[\frac{16TD}{\pi(D^4-d^4)}\right]_{br}$$
$$8000 = \frac{16(0.6701T)(3)}{\pi(3^4-2^4)}$$

$T = 50789.32 \text{ lb} \cdot \text{in}$
$T = 4232.44 \text{ lb} \cdot \text{ft}$

Based on steel core (T_{st} = 0.3299T):

$$\tau_{max} = \left[\frac{16TD}{\pi D^3} \right]_{st}$$

$$12\,000 = \frac{16(0.3299T)}{\pi(2^3)}$$

$T = 57\,137.18 \text{ lb} \cdot \text{in}$
$T = 4761.43 \text{ lb} \cdot \text{ft}$

Use T = 4232.44 lb·ft ***answer***

Problem 318

A solid aluminum shaft 2 in. in diameter is subjected to two torques as shown in Fig. P-318. Determine the maximum shearing stress in each segment and the angle of rotation of the free end. Use G = 4 × 10^6 psi.

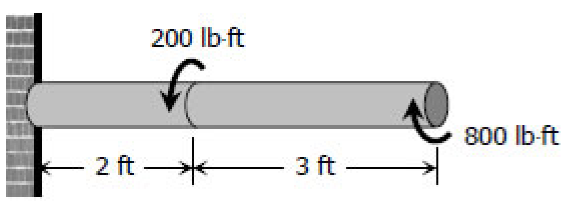
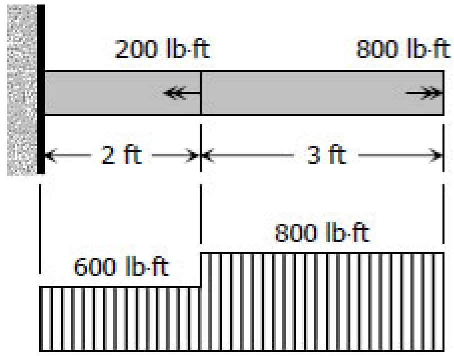


Figure P-318

Solution 318



$$\tau_{max} = \frac{16T}{\pi D^3}$$

For 2-ft segment:

$$\tau_{max2} = \frac{16(600)(12)}{\pi(2^3)} = 4583.66\ \text{psi}$$ ***answer***

For 3-ft segment:

$$\tau_{max3} = \frac{16(800)(12)}{\pi(2^3)} = 6111.55\ \text{psi}$$ ***answer***

$$\theta = \frac{TL}{JG}$$
$$\theta = \frac{1}{JG}\Sigma TL$$
$$\theta = \frac{1}{\frac{1}{31}\pi(2^4)(4\times 10^6)}[\,600(2) + 800(3)\,]\,(12^2)$$
$$\theta = 0.0825\ \text{rad}$$
$$\theta = 4.73^\circ$$ ***answer***

Problem 319

The compound shaft shown in Fig. P-319 is attached to rigid supports. For the bronze segment AB, the diameter is 75 mm, τ ≤ 60 MPa, and G = 35 GPa. For the steel segment BC, the diameter is 50 mm, τ ≤ 80 MPa, and G = 83 GPa. If a = 2 m and b = 1.5 m, compute the maximum torque T that can be applied.

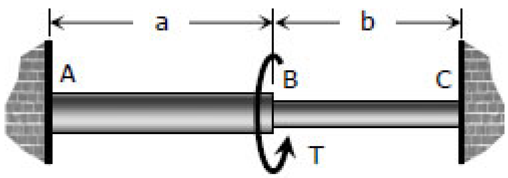
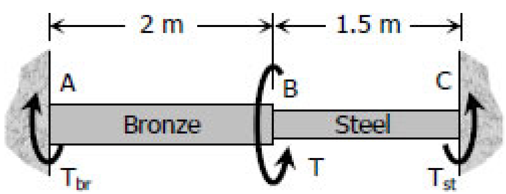


Figure P-319 and P-320

Solution 319



$\Sigma M = 0$
$T = T_{br} + T_{st}$ →**Equation (1)**

$\theta_{br} = \theta_{st}$
$\left(\dfrac{TL}{JG}\right)_{br} = \left(\dfrac{TL}{JG}\right)_{st}$
$\dfrac{T_{br}(2)(1000)}{\frac{1}{32}\pi(75^4)(35\,000)} = \dfrac{T_{st}(1.5)(1000)}{\frac{1}{32}\pi(50^4)(83\,000)}$
$T_{br} = 1.6011T_{st}$ →**Equation (2a)**
$T_{st} = 0.6246T_{br}$ →**Equation (2b)**

$\tau_{max} = \dfrac{16T}{\pi D^3}$

Based on τ_{br} ≤ 60 MPa

$$60 = \frac{16T_{br}}{\pi(75^3)}$$

$$T_{br} = 4\,970\,097.75 \text{ N} \cdot \text{mm}$$

$T_{br} = 4.970 \text{ kN} \cdot \text{m}$ →Maximum allowable torque for bronze

$T_{st} = 0.6246(4.970)$ →From Equation (2b)

$$T_{st} = 3.104 \text{ kN} \cdot \text{m}$$

Based on τ_{br} ≤ 80 MPa

$$80 = \frac{16T_{st}}{\pi(50^3)}$$

$$T_{st} = 1\,963\,495.41 \text{ N} \cdot \text{mm}$$

$T_{st} = 1.963 \text{ kN} \cdot \text{m}$ →Maximum allowable torque for steel

$T_{br} = 1.6011(1.963)$ →From Equation (2a)

$$T_{br} = 3.142 \text{ kN} \cdot \text{m}$$

Use T_{br} = 3.142 kN·m and T_{st} = 1.963 kN·m

$T = 3.142 + 1.963$ →From Equation (1)

$T = 5.105 \text{ kN} \cdot \text{m}$***answer***

Problem 320

In Prob. 319, determine the ratio of lengths b/a so that each material will be stressed to its permissible limit. What torque T is required?

Solution 320

From Solution 319:
Maximum T_{br} = 4.970 kN·m
Maximum T_{st} = 1.963 kN·m

$$\theta_{br} = \theta_{st}$$
$$\left(\frac{TL}{JG}\right)_{br} = \left(\frac{TL}{JG}\right)_{st}$$
$$\frac{4.973a(1000^2)}{\frac{1}{32}\pi(75^4)(35\,000)} = \frac{1.963b(1000^2)}{\frac{1}{32}\pi(50^4)(83\,000)}$$
$$b/a = 1.187$$

$$T = T_{br\ max} + T_{st\ max}$$
$$T = 4.970 + 1.963$$
$$T = 6.933\ \mathrm{kN\cdot m}$$ ***answer***

Problem 321

A torque T is applied, as shown in Fig. P-321, to a solid shaft with built-in ends. Prove that the resisting torques at the walls are T_1 = Tb/L and T_2 = Ta/L. How would these values be changed if the shaft were hollow?

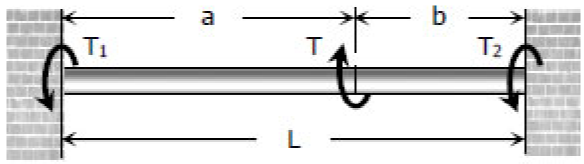


Figure P-321

Solution 321

$\Sigma M = 0$
$T = T_1 + T_2$ → **Equation (1)**

$\theta_1 = \theta_2$
$\left(\frac{TL}{JG}\right)_1 = \left(\frac{TL}{JG}\right)_2$
$\frac{T_1 a}{JG} = \frac{T_2 b}{JG}$
$T_1 = \frac{b}{a} T_2$ → **Equation (2a)**
$T_2 = \frac{a}{b} T_1$ → **Equation (2b)**

Equations (1) and (2b):
$T = T_1 + \frac{a}{b} T_1$
$T = \frac{T_1 b + T_1 a}{b}$
$T = \frac{(b + a) T_1}{b}$
$T = \frac{L T_1}{b}$
$T_1 = Tb/L$ ***ok!***

Equations (1) and (2a):
$T = \frac{b}{a} T_2 + T_2$
$T = \frac{T_2 b + T_2 a}{a}$

$$T = \frac{(b+a)T_2}{a}$$
$$T = \frac{LT_2}{a}$$
$$T_2 = Ta/L$$ ***ok!***

If the shaft were hollow, Equation (1) would be the same and the equality $\theta_1 = \theta_2$, by direct investigation, would yield the same result in Equations (2a) and (2b). Therefore, the values of T_1 and T_2 are the same (**no change**) if the shaft were hollow.

Problem

A solid steel shaft is loaded as shown in Fig. P-322. Using G = 83 GPa, determine the required diameter of the shaft if the shearing stress is limited to 60 MPa and the angle of rotation at the free end is not to exceed 4 deg.

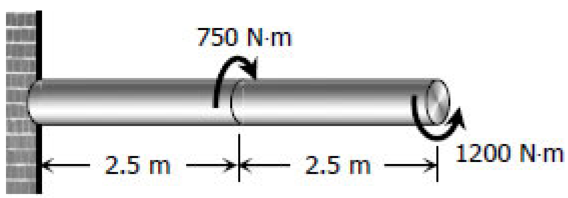
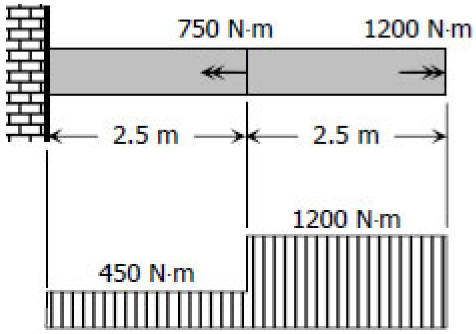


Figure P-322

Solution

Based on maximum allowable shear:

$$\tau_{max} = \frac{16T}{\pi D^3}$$



For the 1st segment:

$$60 = \frac{450(2.5)(1000^2)}{\pi D^3}$$

$$D = 181.39 \text{ mm}$$

For the 2nd segment:

$$60 = \frac{1200(2.5)(1000^2)}{\pi D^3}$$
$$D = 251.54 \text{ mm}$$

Based on maximum angle of twist:

$$\theta = \frac{TL}{JG}$$
$$\theta = \frac{1}{JG}\Sigma TL$$
$$4^\circ\left(\frac{\pi}{180^\circ}\right) = \frac{1}{\frac{1}{32}\pi D^4(83\,000)}\left[\,450(2.5) + 1200(2.5)\,\right](1000^2)$$
$$D = 51.89 \text{ mm}$$

Use **D = 251.54 mm *answer***

Problem 323

A shaft composed of segments AC, CD, and DB is fastened to rigid supports and loaded as shown in Fig. P-323. For bronze, G = 35 GPa; aluminum, G = 28 GPa, and for steel, G = 83 GPa. Determine the maximum shearing stress developed in each segment.

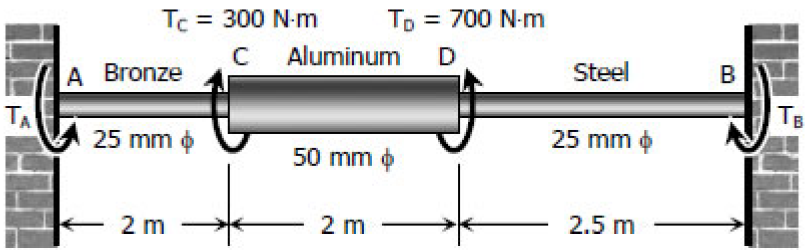
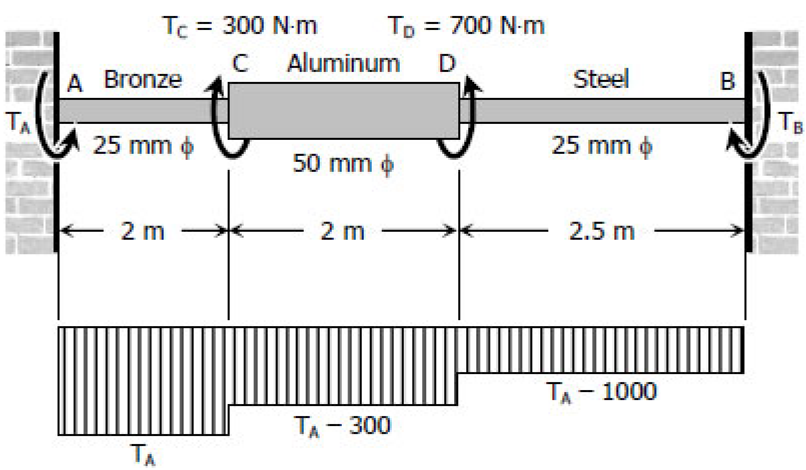


Figure P-323

Solution 323

Stress developed in each segment with respect to T_A:



The rotation of B relative to A is zero.

$$\theta_{A/B} = 0$$

$$\left(\Sigma \frac{TL}{JG}\right)_{A/B} = 0$$

$$\frac{T_A(2)(1000^2)}{\frac{1}{32}\pi(25^4)(35\,000)} + \frac{(T_A - 300)(2)(1000^2)}{\frac{1}{32}\pi(50^4)(28\,000)} + \frac{(T_A - 1000)(2.5)(1000^2)}{\frac{1}{32}\pi(25^4)(83\,000)} = 0$$

$$\frac{2T_A}{(25^4)(35)} + \frac{2(T_A - 300)}{(50^4)(28)} + \frac{2.5(T_A - 1000)}{(25^4)(83)} = 0$$

$$\frac{16T_A}{35} + \frac{T_A - 300}{28} + \frac{20(T_A - 1000)}{83} = 0$$

$$\frac{16}{25}T_A + \frac{1}{28}T_A - \frac{75}{7} + \frac{20}{83}T_A - \frac{20\,000}{83} = 0$$

$$\frac{8527}{11\,620}T_A = 251.678$$

$$T_A = 342.97 \text{ N} \cdot \text{m}$$

$$\Sigma M = 0$$
$$T_A + T_B = 300 + 700$$
$$342.97 + T_B = 1000$$
$$T_B = 657.03 \text{ N} \cdot \text{m}$$

$$T_{br} = 342.97 \text{ N} \cdot \text{m}$$
$$T_{al} = 342.97 - 300 = 42.97 \text{ N} \cdot \text{m}$$
$$T_{st} = 342.97 - 1000 = -657.03 \text{ N} \cdot \text{m} = -T_B$$ ***(ok!)***

$$\tau_{max} = \frac{16T}{\pi D^3}$$
$$\tau_{br} = \frac{16(342.97)(1000)}{\pi(25^3)} = 111.79 \text{ MPa}$$ ***answer***
$$\tau_{al} = \frac{16(42.97)(1000)}{\pi(50^3)} = 1.75 \text{ MPa}$$ ***answer***
$$\tau_{st} = \frac{16(657.03)(1000)}{\pi(25^3)} = 214.16 \text{ MPa}$$ ***answer***

Problem 324

The compound shaft shown in Fig. P-324 is attached to rigid supports. For the bronze segment AB, the maximum shearing stress is limited to 8000 psi and for the steel segment BC, it is limited to 12 ksi. Determine the diameters of each segment so that each material will be simultaneously stressed to its permissible limit when a torque T = 12 kip·ft is applied. For bronze, G = 6 × 10^6 psi and for steel, G = 12 × 10^6 psi.

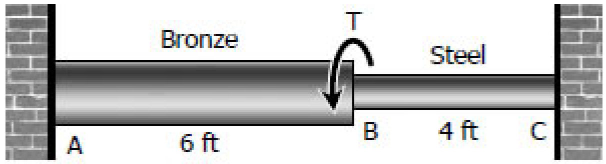


Figure P-324

Solution 324

$$\tau_{max} = \frac{16T}{\pi D^3}$$

For bronze:

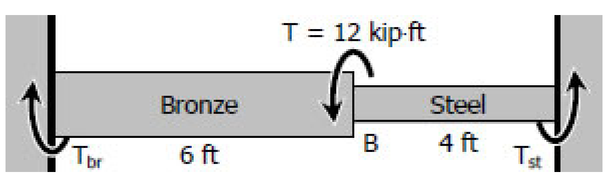
$$8000 = \frac{16T_{br}}{\pi D_{br}^3}$$

$$T_{br} = 500\pi D_{br}^3 \text{ lb} \cdot \text{in}$$

For steel:

$$12\,000 = \frac{16T_{st}}{\pi D_{st}^3}$$

$$T_{st} = 750\pi D_{br}^3 \text{ lb} \cdot \text{in}$$



$$\Sigma M = 0$$

$$T_{br} + T_{st} = T$$

$$T_{br} + T_{st} = 12(1000)(12)$$

$$T_{br} + T_{st} = 144\,000 \text{ lb} \cdot \text{in}$$

$$500\pi D_{br}^3 + 750\pi D_{br}^3 = 144\,000$$

$$D_{br}^3 = 288/\pi + 1.5D_{st}^3 \rightarrow$$ ***Equation (1)***

$$\theta_{br} = \theta_{st}$$

$$\left(\frac{TL}{JG}\right)_{br} = \left(\frac{TL}{JG}\right)_{st}$$

$$\frac{T_{br}(6)}{\frac{1}{32}\pi D_{br}^4(6 \times 10^6)} = \frac{T_{st}(4)}{\frac{1}{32}\pi D_{st}^4(12 \times 10^6)}$$

$$\frac{T_{br}}{D_{br}^4} = \frac{T_{st}}{3D_{st}^4}$$

$$\frac{500\pi D_{br}^3}{D_{br}^4} = \frac{750\pi D_{st}^3}{3D_{st}^4}$$

$$D_{st} = 0.5D_{br}$$

From Equation (1)

$$D_{br}^3 = 288/\pi - 1.5(0.5D_{br})^3$$

$$D_{br} = 288/\pi$$

$$D_{br} = 4.26 \text{ in.}$$ ***answer***

$$D_{st} = 0.5(4.26) = 2.13 \text{ in.}$$ ***answer***

Problem 325

The two steel shaft shown in Fig. P-325, each with one end built into a rigid support have flanges rigidly attached to their free ends. The shafts are to be bolted together at their flanges. However, initially there is a 6° mismatch in the location of the bolt holes as shown in the figure. Determine the maximum shearing stress in each shaft after the shafts are bolted together. Use G = 12×10^6 psi and neglect deformations of the bolts and flanges.

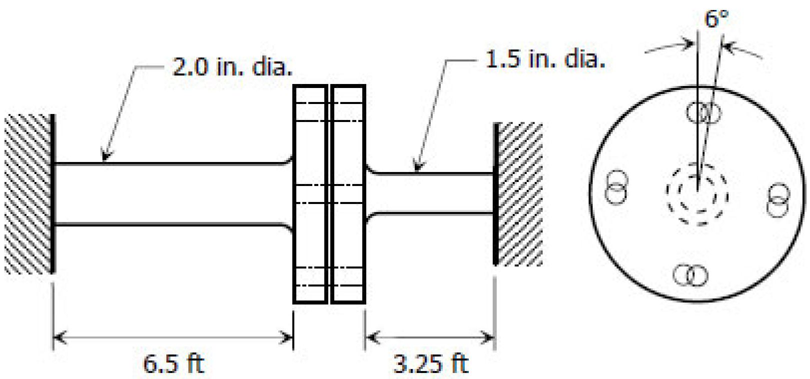


Figure P-325

Solution 325

$$\theta_{of\ 6.5'\ shaft} + \theta_{of\ 3.25'\ shaft} = 6^\circ$$

$$\left(\frac{TL}{JG}\right)_{of\ 6.5'\ shaft} + \left(\frac{TL}{JG}\right)_{of\ 3.25'\ shaft} = 6^\circ\left(\frac{\pi}{180^\circ}\right)$$

$$\frac{T(6.5)(12^2)}{\frac{1}{32}\pi(2^4)(12\times10^6)} + \frac{T(3.25)(12^2)}{\frac{1}{32}\pi(1.5^4)(12\times10^6)} = \frac{\pi}{30}$$

$$T = 817.32\ \mathrm{lb\cdot ft}$$

$$\tau_{max} = \frac{16T}{\pi D^3}$$

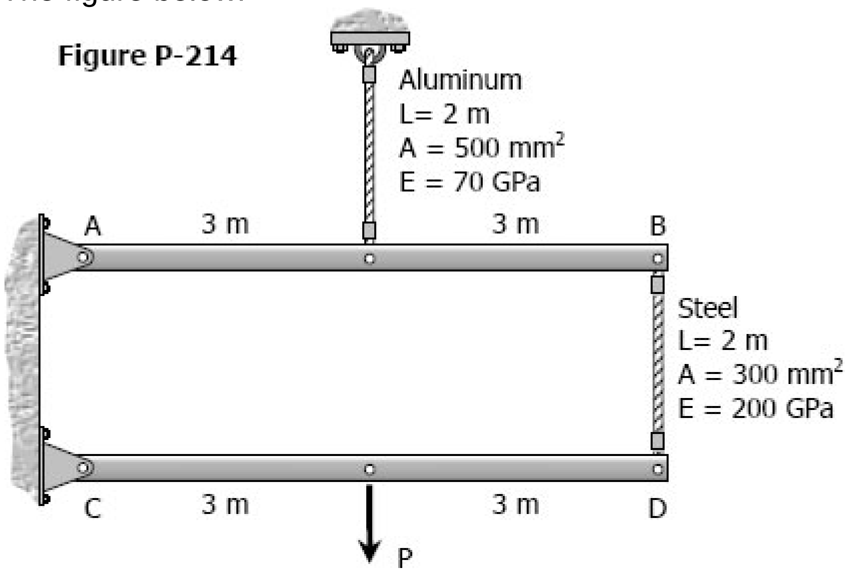
$$\tau_{of\ 6.5'\ shaft} = \frac{16(817.32)(12)}{\pi(2^3)} = 6243.86\ \mathrm{psi}$$ ***answer***

$$\tau_{of\ 3.25'\ shaft} = \frac{16(817.32)(12)}{\pi(1.5^3)} = 14\,800.27\ \mathrm{psi}$$ ***answer***

Problem 214 page 41

Given:
Maximum vertical movement of P = 5 mm

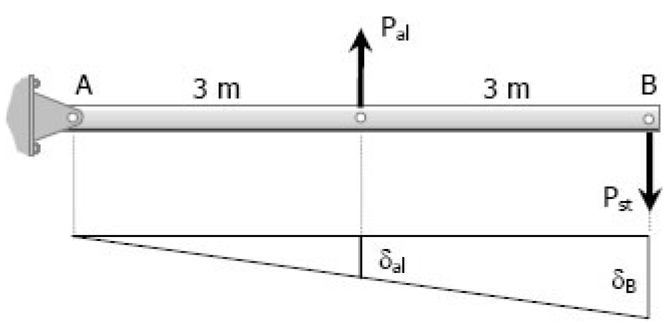
The figure below:



Required: The maximum force P that can be applied neglecting the weight of all members.

Solution 41

Member AB:



FBD and movement diagram of bar AB

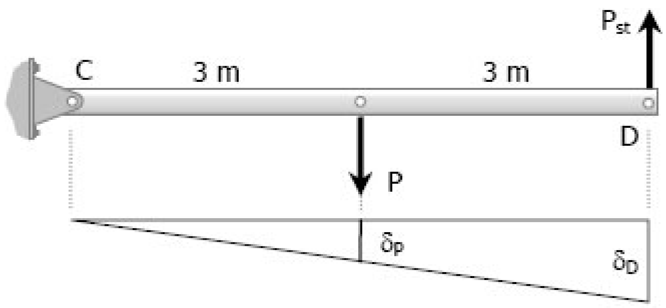
$$\Sigma M_A = 0$$
$$3P_{al} = 6P_{st}$$
$$P_{al} = 2P_{st}$$

By ratio and proportion:

$$\frac{\delta_B}{6} = \frac{\delta_{al}}{3}$$
$$\delta_B = 2\delta_{al} = 2\left[\frac{PL}{AE}\right]_{al}$$
$$\delta_B = 2\left[\frac{P_{al}\,(2000)}{500(70\,000)}\right]$$

$\delta_B = \frac{1}{8750} P_{al} = \frac{1}{8750}(2P_{st})$
$\delta_B = \frac{1}{4375} P_{st}$ → movement of B

Member CD:



FBD and movement diagram of bar CD

Movement of D:

$$\delta_D = \delta_{st} + \delta_B = \left[\frac{PL}{AE}\right]_{st} + \frac{1}{4375} P_{st}$$
$$\delta_D = \frac{P_{st}\,(2000)}{300(200\,000)} + \frac{1}{4375} P_{st}$$
$$\delta_D = \frac{11}{42\,000} P_{st}$$

$$\Sigma M_C = 0$$
$$6P_{st} = 3P$$
$$P_{st} = \tfrac{1}{2} P$$

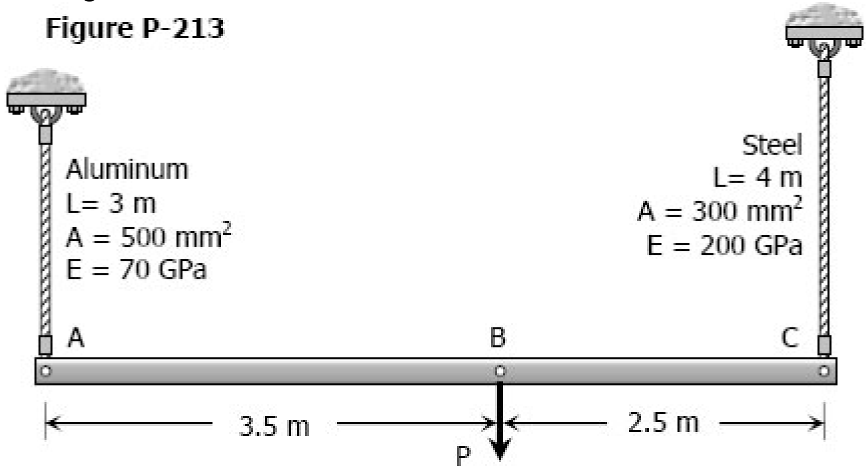
By ratio and proportion:

$$\frac{\delta_P}{3} = \frac{\delta_D}{6}$$
$$\delta_P = \tfrac{1}{2}\delta_D = \tfrac{1}{2}\left(\tfrac{11}{42\,000} P_{st}\right)$$
$$\delta_P = \frac{11}{84\,000} P_{st}$$
$$5 = \frac{11}{84\,000}\left(\tfrac{1}{2}P\right)$$
$P = 76\,363.64\ \mathrm{N} = 76.4\ \mathrm{kN}$ →***answer***

Problem 213 page 41

Given:
Rigid bar is horizontal before P = 50 kN is applied

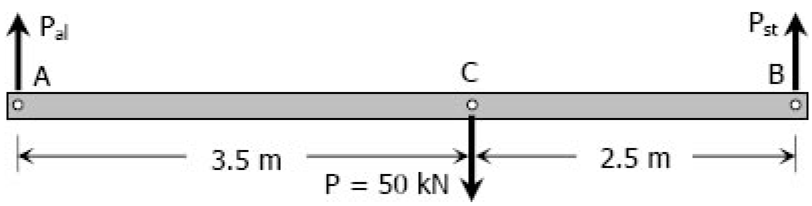
The figure below:



Required: Vertical movement of P

Solution 213

Free body diagram:



For aluminum:

$$\Sigma M_B = 0$$
$$6P_{al} = 2.5(50)$$
$$P_{al} = 20.83\,\text{kN}$$

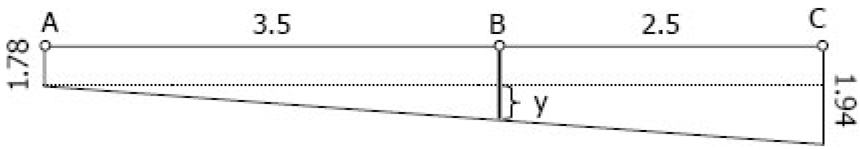
$$\delta = \frac{PL}{AE}$$
$$\delta_{al} = \frac{20.83(3)1000^2}{500(70\,000)}$$
$$\delta_{al} = 1.78\,\text{mm}$$

For steel:

$$\Sigma M_A = 0$$
$$6P_{st} = 3.5(50)$$
$$P_{st} = 29.17\,\text{kN}$$

$$\delta = \frac{PL}{AE}$$
$$\delta_{st} = \frac{29.17(4)1000^2}{300(200\,000)}$$
$$\delta_{st} = 1.94 \text{ mm}$$

Movement diagram:



$$\frac{y}{3.5} = \frac{1.94 - 1.78}{6}$$
$$y = 0.09 \text{ mm}$$
$$\delta_B = \text{vertical movement of } P$$
$$\delta_B = 1.78 + y = 1.78 + 0.09$$
$\delta_B = 1.87 \text{ mm}$ →***answer***

Problem 211 page 40

Given:
Maximum overall deformation = 3.0 mm
Maximum allowable stress for steel = 140 MPa
Maximum allowable stress for bronze = 120 MPa
Maximum allowable stress for aluminum = 80 MPa
E_{st} = 200 GPa
E_{al} = 70 GPa
E_{br} = 83 GPa
The figure below:

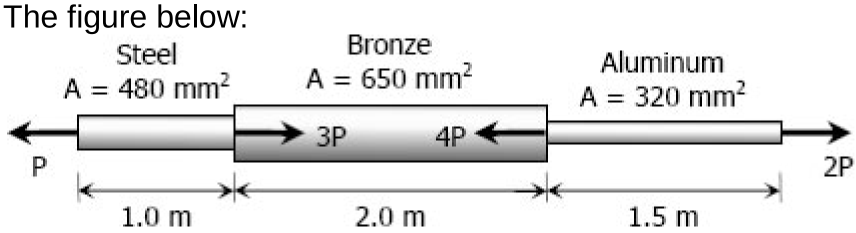
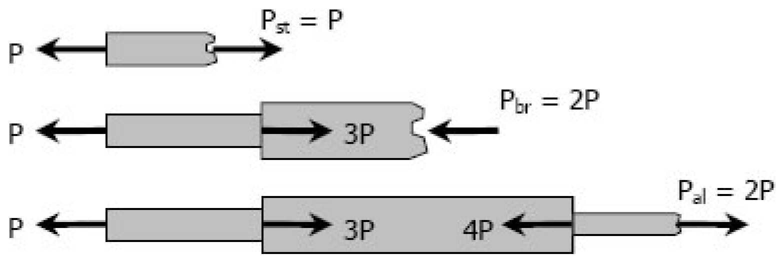


Figure P -211

Required: The largest value of P

Solution 211



Based on allowable stresses:

Steel:

$$P_{st} = \sigma_{st}\,A_{st}$$
$$P = 140(480) = 67\,200\text{ N}$$
$$P = 67.2\text{ kN}$$

Bronze:

$$P_{br} = \sigma_{br}\,A_{br}$$
$$2P = 120(650) = 78\,000\text{ N}$$
$$P = 39\,000\text{ N} = 39\text{ kN}$$

Aluminum:

$$P_{al} = \sigma_{al}\,A_{al}$$
$$2P = 80(320) = 25\,600\text{ N}$$
$$P = 12\,800\text{ N} = 12.8\text{ kN}$$

Based on allowable deformation:
(*steel and aluminum lengthens, bronze shortens*)

$$\delta = \delta_{st} - \delta_{br} + \delta_{al}$$
$$3 = \frac{P(1000)}{480(200\,000)} - \frac{2P(2000)}{650(70\,000)} + \frac{2P(1500)}{320(83\,000)}$$
$$3 = \left(\tfrac{1}{96\,000} - \tfrac{1}{11\,375} + \tfrac{3}{26\,560}\right)P$$
$$P = 84\,610.99\text{ N} = 84.61\text{ kN}$$

Use the smallest value of P, **P = 12.8 kN**

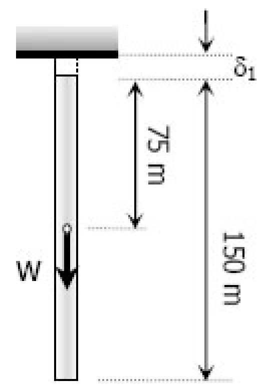
Problem 206 page 39

Given:
Cross-sectional area = 300 mm^2
Length = 150 m
tensile load at the lower end = 20 kN
Unit mass of steel = 7850 kg/m^3
E = 200 × 10^3 MN/m^2

Required: Total elongation of the rod

Solution 206

Elongation due to its own weight:



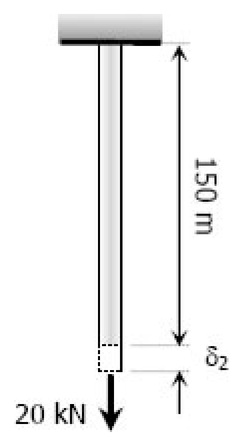
$$\delta_1 = \frac{PL}{AE}$$

Where:
P = W = 7850(1/1000)3(9.81)[300(150)(1000)]
P = 3465.3825 N
L = 75(1000) = 75 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_1 = \frac{3\,465.3825(75\,000)}{300(200\,000)}$$

δ_1= 4.33 mm

Elongation due to applied load:



$$\delta_2 = \frac{PL}{AE}$$

Where:
P = 20 kN = 20 000 N
L = 150 m = 150 000 mm
A = 300 mm^2
E = 200 000 MPa

$$\delta_2 = \frac{20\,000(150\,000)}{300(200\,000)}$$

δ_2= 50 mm

Total elongation:

$\delta = \delta_1 + \delta_2$
$\delta = 4.33 + 50 = 54.33\ \mathrm{mm}$ →***answer***

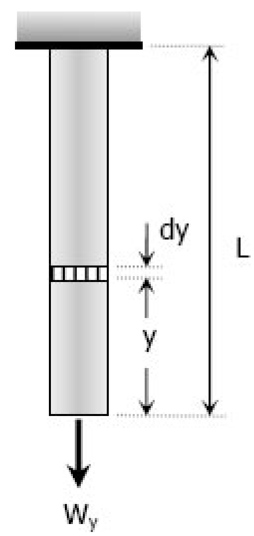
Problem 205 page 39

Given:
Length of bar = L
Cross-sectional area = A
Unit mass = ρ
The bar is suspended vertically from one end

Required:
Show that the total elongation $\delta = \rho g L^2 / 2E$.
If total mass is M, show that $\delta = MgL/2AE$

Solution 205

$$\delta = \frac{PL}{AE}$$



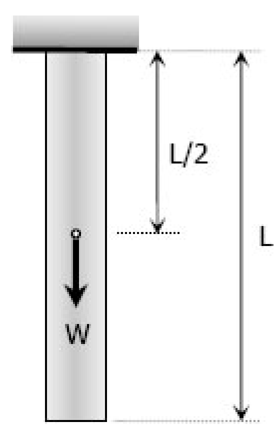
From the figure:
$\delta = d\delta$
$P = Wy = (\rho Ay)g$
$L = dy$

$$d\delta = \frac{(\rho Ay)g\,dy}{AE}$$
$$\delta = \frac{\rho g}{E}\int_0^L y\,dy = \frac{\rho g}{E}\left[\frac{y^2}{2}\right]_0^L$$
$$\delta = \frac{\rho g}{2E}[\,L^2 - 0^2\,]$$
$$\delta = \rho g L^2/2E$$ ***ok!***

Given the total mass M
$$\rho = M/V = M/AL$$

$$\delta = \frac{\rho g L^2}{2E} = \frac{\frac{M}{AL}\cdot gL^2}{2E}$$
$$\delta = \frac{MgL}{2AE}$$ ***ok!***

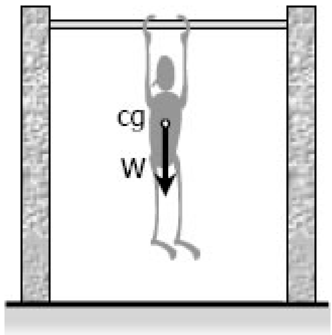
Another Solution:



$$\delta = \frac{PL}{AE}$$

Where:
P = W = (ρAL)g
L = L/2

$$\delta = \frac{[(\rho AL)g](L/2)}{AE}$$
$\delta = \rho g L^2/2E$ ***ok!***



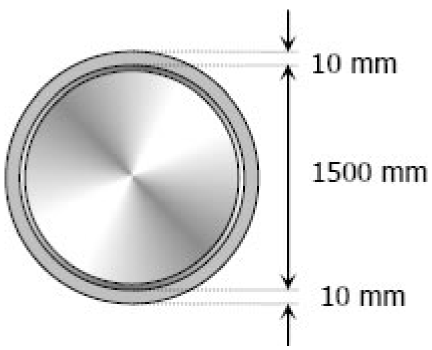
For you to feel the situation, position yourself in pull-up exercise with your hands on the bar and your body hang freely above the ground. Notice that your arms suffer all your weight and your lower body fells no stress (center of weight is approximately just below the chest). If your body is the bar, the elongation will occur at the upper half of it.

Problem 208 page 40

Given:
Thickness of steel tire = 100 mm
Width of steel tire = 80 mm
Inside diameter of steel tire = 1500.0 mm
Diameter of steel wheel = 1500.5 mm
Coefficient of static friction = 0.30
E = 200 GPa

Required: Torque to twist the tire relative to the wheel

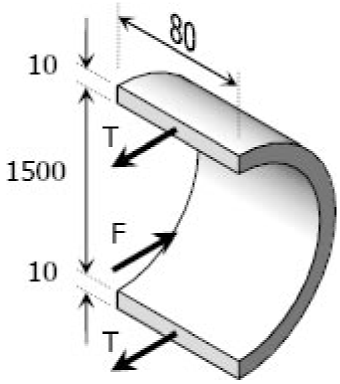
Solution 208



$$\delta = \frac{PL}{AE}$$

Where:
$\delta = \pi\ (1500.5 - 1500) = 0.5\pi$ mm
P = T
L = 1500π mm
A = 10(80) = 800 mm^2
E = 200 000 MPa

$$0.5\pi = \frac{T(1500\pi)}{800(200\,000)}$$
$$T = 53\,333.33\ \mathrm{N}$$



$F = 2T$
$p(1500)(80) = 2(53\,333.33)$
$p = 0.8889\ \mathrm{MPa}$ →internal pressure

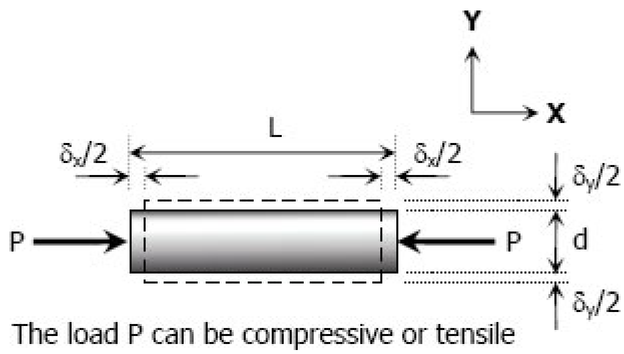
Total normal force, N:
N = p × contact area between tire and wheel
N = 0.8889 × π(1500.5)(80)
N = 335 214.92 N

Friction resistance, f:
f = μN = 0.30(335 214.92)
f = 100 564.48 N = 100.56 kN
Torque = f × ½(diameter of wheel)
Torque = 100.56 × 0.75025
Torque = **75.44 kN · m**

Problem 222

A solid cylinder of diameter d carries an axial load P. Show that its change in diameter is $4P\nu$ / πEd.

Solution 222



$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$
$$\varepsilon_y = -\nu\,\varepsilon_x$$
$$\varepsilon_y = -\nu\,\frac{\sigma_x}{E}$$
$$\frac{\delta_y}{d} = -\nu\,\frac{-P}{AE}$$
$$\delta_y = \frac{Pd}{\frac{1}{4}\pi d^2\,E}$$
$$\delta_y = \frac{4P\nu}{\pi Ed} \rightarrow$$ ***ok***

Problem 223 page 44

Given:
Dimensions of the block:
- x direction = 3 inches
- y direction = 2 inches
- z direction = 4 inches

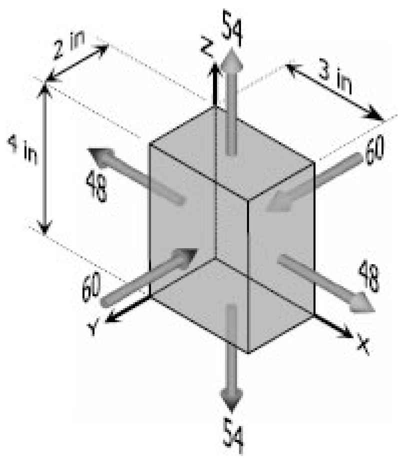
Triaxial loads in the block
- x direction = 48 kips tension
- y direction = 60 kips compression
- z direction = 54 kips tension

Poisson's ratio, ν = 0.30

Modulus of elasticity, E = 29 × 10^6 psi

Required:
Single uniformly distributed load in the x direction that would produce the same deformation in the y direction as the original loading.

Solution 223



For triaxial deformation (tensile triaxial stresses):
(compressive stresses are negative stresses)
$$\varepsilon_y = \frac{1}{E}[\,\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\sigma_x = \frac{P_x}{A_{yz}} = \frac{48}{4(2)} = 6.0\,\text{ksi}$$ (tension)
$$\sigma_y = \frac{P_y}{A_{xz}} = \frac{60}{4(3)} = 5.0\,\text{ksi}$$ (compression)
$$\sigma_z = \frac{P_z}{A_{xy}} = \frac{54}{2(3)} = 9.0\,\text{ksi}$$ (tension)

$$\varepsilon_y = \frac{1}{29 \times 10^6}[\,-5000 - 0.30(6000 + 9000)]$$
$$\varepsilon_y = -3.276 \times 10^{-4}$$

$\varepsilon_y$is negative, thus tensile force is required in the x-direction to produce the same deformation in the y-direction as the original forces.

For equivalent single force in the x-direction:
(uniaxial stress)
$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}$$
$$-\nu\varepsilon_x = \varepsilon_y$$
$$-\nu\frac{\sigma_x}{E} = \varepsilon_y$$

$$-0.30\left(\frac{\sigma_x}{29\times10^6}\right) = -3.276\times10^{-4}$$
$$\sigma_x = 31\,666.67\text{ psi}$$
$$\sigma_x = \frac{P_x}{4(2)} = 31\,666.67$$
$P_x = 253\,333.33\text{ lb}$ (tension)
$P_x = 253.33\text{ kips}$ (tension) → ***answer***

Problem 224

For the block loaded triaxially as described in Prob. 223, find the uniformly distributed load that must be added in the x direction to produce no deformation in the z direction.

Solution 224

$$\varepsilon_z = \frac{1}{E}[\,\sigma_z - \nu(\sigma_x + \sigma_y)\,]$$

Where
σ_x = 6.0 ksi (tension)
σ_y = 5.0 ksi (compression)
σ_z = 9.0 ksi (tension)

$$\varepsilon_z = \frac{1}{29\times10^6}[\,9000 - 0.3(6000-5000)\,]$$
$$\varepsilon_z = 2.07\times10^{-5}$$
ε_z is positive, thus positive stress is needed in the x-direction to eliminate deformation in z-direction.

The application of loads is still simultaneous:
(*No deformation means zero strain*)
$$\varepsilon_z = \frac{1}{E}[\,\sigma_z - \nu(\sigma_x + \sigma_y)\,] = 0$$
$$\sigma_z = \nu(\sigma_x + \sigma_y)$$
σ_y = 5.0 ksi (compression)
σ_z = 9.0 ksi (tension)
$$9000 = 0.30(\sigma_x - 5000)$$
$$\sigma_x = 35\,000\text{ psi}$$

$$\sigma_{added} + 6000 = 35\,000$$
$$\sigma_{added} = 29\,000\text{ psi}$$
$$\frac{P_{added}}{2(4)} = 29\,000$$
$$P_{added} = 232\,000\text{ lb}$$
$P_{added} = 232\text{ kips}$ → ***answer***

Problem 225

A welded steel cylindrical drum made of a 10-mm plate has an internal diameter of 1.20 m. Compute the change in diameter that would be caused by an internal pressure of 1.5 MPa. Assume that Poisson's ratio is 0.30 and E = 200 GPa.

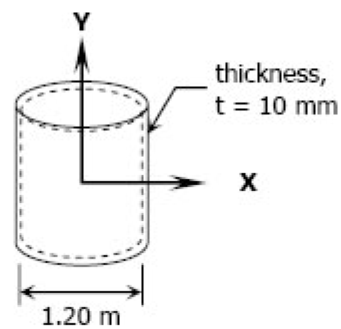
Solution 225

σ_y = longitudinal stress

$$\sigma_y = \frac{pD}{4t} = \frac{1.5(1200)}{4(10)}$$
$$\sigma_y = 45\text{ MPa}$$

σ_x = tangential stress

$$\sigma_y = \frac{pD}{2t} = \frac{1.5(1200)}{2(10)}$$
$$\sigma_y = 90\text{ MPa}$$



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$
$$\varepsilon_x = \frac{90}{200\,000} - 0.3\left(\frac{45}{200\,000}\right)$$
$$\varepsilon_x = 3.825 \times 10^{-4}$$

$$\varepsilon_x = \frac{\Delta D}{D}$$
$$\Delta D = \varepsilon_x\, D == (3.825 \times 10^{-4})(1200)$$
$$\Delta D = 0.459\text{ mm}$$ →***answer***

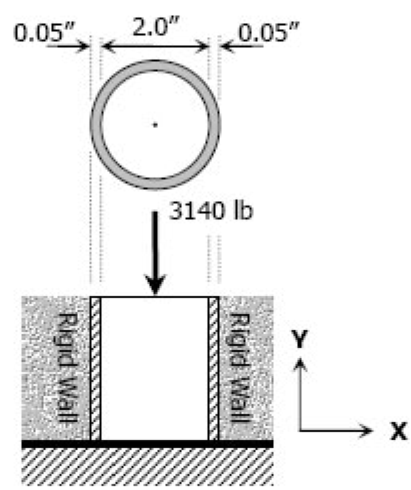
Problem 226

A 2-in.-diameter steel tube with a wall thickness of 0.05 inch just fits in a rigid hole. Find the tangential stress if an axial compressive load of 3140 lb is applied. Assume ν = 0.30 and neglect the possibility of buckling.

Solution 226

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$

where
σ_x = tangential stress
σ_y = longitudinal stress



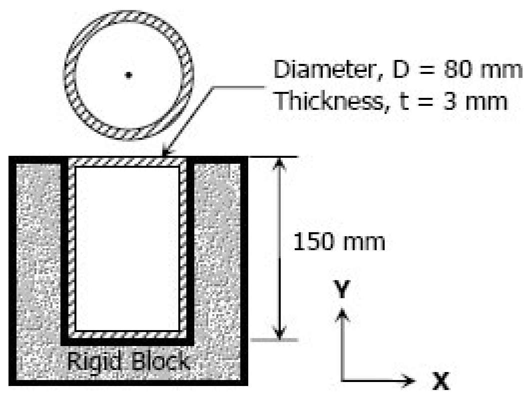
$\sigma_y = P_y / A = 3140 / (\pi \times 2 \times 0.05)$
$\sigma_y = 31{,}400/\pi$ psi

$$\sigma_x = 0.30(31400/\pi)$$
$$\sigma_x = 9430/\pi \text{ psi}$$
$$\sigma_x = 2298.5 \text{ psi}$$

Problem 227

A 150-mm-long bronze tube, closed at its ends, is 80 mm in diameter and has a wall thickness of 3 mm. It fits without clearance in an 80-mm hole in a rigid block. The tube is then subjected to an internal pressure of 4.00 MPa. Assuming $\nu = 1/3$ and $E = 83$ GPa, determine the tangential stress in the tube.

Solution 227



Longitudinal stress:

$$\sigma_y = \frac{pD}{4t} = \frac{4(80)}{4(3)}$$
$$\sigma_y = \frac{80}{3} \text{ MPa}$$

The **strain in the x-direction** is:

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} = 0$$
$\sigma_x = \nu\sigma_y$ = tangential stress
$$\sigma_x = \frac{1}{3}\left(\frac{80}{3}\right)$$
$\sigma_x = 8.89 \text{ MPa}$ → ***answer***

Problem 228

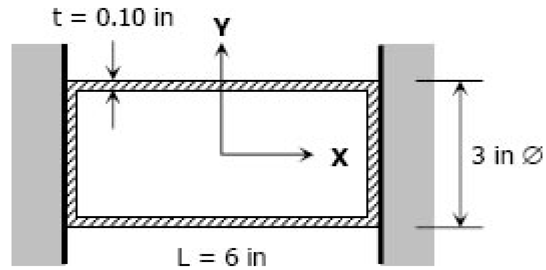
A 6-in.-long bronze tube, with closed ends, is 3 in. in diameter with a wall thickness of 0.10 in. With no internal pressure, the tube just fits between two rigid end walls. Calculate the longitudinal and tangential stresses for an internal pressure of 6000 psi. Assume $\nu = 1/3$ and $E = 12 \times 10^6$ psi.

Solution 228

$\varepsilon = \dfrac{\sigma_x}{E} - \nu \dfrac{\sigma_y}{E} = 0$
$\sigma_x = \nu\,\sigma_y = \sigma_l$ →longitudinal stress
$\sigma_t = \sigma_y$ →tangential stress

$\sigma_t = \dfrac{pD}{2t} = \dfrac{6000(3)}{2(0.10)}$
$\sigma_t = 90\,000\ \mathrm{psi}$ →***answer***

$\sigma_l = \nu\,\sigma_y = \frac{1}{3}(90\,000)$
$\sigma_l = 30\,000\ \mathrm{psi}$ →***answer***

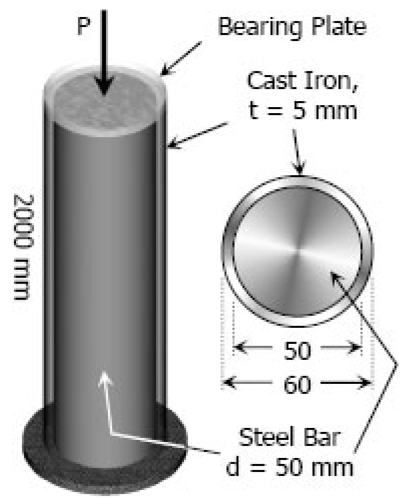


Problem 233

A steel bar 50 mm in diameter and 2 m long is surrounded by a shell of a cast iron 5 mm thick. Compute the load that will compress the combined bar a total of 0.8 mm in the length of 2 m. For steel, E = 200 GPa, and for cast iron, E = 100 GPa.

Solution 233

$$\delta = \frac{PL}{AE}$$
$$\delta = \delta_{cast\ iron} = \delta_{steel} = 0.8\ \mathrm{mm}$$



$$\delta_{cast\ iron} = \frac{P_{cast\ iron}(2000)}{[\,\frac{1}{4}\pi(60^2 - 50^2)\,](100\,000)} = 0.8$$
$$P_{cast\ iron} = 11\,000\pi\ \mathrm{N}$$

$$\delta_{steel} = \frac{P_{steel}(2000)}{[\,\frac{1}{4}\pi(50^2)\,](200\,000)} = 0.8$$
$$P_{steel} = 50\,000\pi\ \mathrm{N}$$

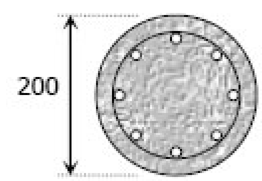
$$\Sigma F_V = 0$$
$$P = P_{cast\ iron} + P_{steel}$$
$$P = 11\,000\pi + 50\,000\pi$$
$$P = 61\,000\pi\ \mathrm{N}$$
$P = 191.64\ \mathrm{kN}$ →***answer***

Problem 234

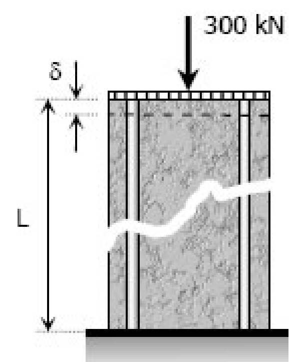
A reinforced concrete column 200 mm in diameter is designed to carry an axial compressive load of 300 kN. Determine the required area of the reinforcing steel if the allowable stresses are 6 MPa and 120 MPa for the concrete and steel, respectively. Use E_{co} = 14 GPa and E_{st} = 200 GPa.

Solution 234

$$\delta_{co} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{co} = \left(\frac{PL}{AE}\right)_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co} L}{14000} = \frac{\sigma_{st} L}{200\,000}$$
$$100\sigma_{co} = 7\sigma_{st}$$



When $\sigma_{st} = 120\ \mathrm{MPa}$
$100\sigma_{co} = 7(120)$
$\sigma_{co} = 8.4\ \mathrm{MPa} > 6\ \mathrm{MPa}$(not ok!)



When $\sigma_{co} = 6\ \mathrm{MPa}$
$100(6) = 7\sigma_{st}$
$\sigma_{st} = 85.71\ \mathrm{MPa}lt; 120\ \mathrm{MPa}$(ok!)

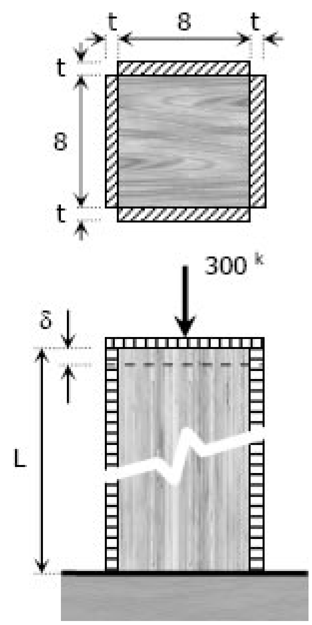
Use $\boldsymbol{\sigma_{co}}$ **= 6 MPa** and $\boldsymbol{\sigma_{st}}$ **= 85.71 MPa**

$$\Sigma F_V = 0$$
$$P_{st} + P_{co} = 300$$
$$\sigma_{st} A_{st} + \sigma_{co} A_{co} = 300$$
$$85.71 Ast + 6\,[\,\tfrac{1}{4}\pi(200)^2 - A_{st}\,] = 300(1000)$$
$$79.71 A_{st} + 60\,000\pi = 300\,000$$
$A_{st} = 1398.9\ \mathrm{mm}^2$ → ***answer***

Problem 235

A timber column, 8 in. × 8 in. in cross section, is reinforced on each side by a steel plate 8 in. wide and t in. thick. Determine the thickness t so that the column will support an axial load of 300 kips without exceeding a maximum timber stress of 1200 psi or a maximum steel stress of 20 ksi. The moduli of elasticity are 1.5 × 10^6 psi for timber, and 29 × 10^6 psi for steel.

Solution 235



$$\delta_{steel} = \delta_{timber}$$
$$\left(\frac{\sigma L}{E}\right)_{steel} = \left(\frac{\sigma L}{E}\right)_{timber}$$
$$\frac{\sigma_{steel} L}{29 \times 10^6} = \frac{\sigma_{bronze} L}{1.5 \times 10^6}$$
$$1.5\sigma_{steel} = 29\sigma_{timber}$$

When σ_{timber} = 1200 psi

$$1.5\sigma_{steel} = 29(1200)$$
$$\sigma_{steel} = 23\,200 \ \text{psi} \ = 23.2 \ \text{ksi} \ > 20 \ \text{ksi}$$ (not ok!)

When σ_{steel} = 20 ksi

$$1.5(20 \times 1000) = 29\sigma_{timber}$$
$$\sigma_{timber} = 1034.48 \ \text{psi} \ lt; 1200 \ \text{psi}$$ (ok!)

Use **σ_{steel} = 20 ksi** and **σ_{timber} = 1.03 ksi**

$$\Sigma F_V = 0$$
$$F_{steel} + F_{timber} = 300$$
$$(\sigma A)_{steel} + (\sigma A)_{timber} = 300$$
$$20[4(8t)] + 1.03(82) = 300$$
$$t = 0.365 \ \text{in}$$ →***answer***

Problem 236

A rigid block of mass M is supported by three symmetrically spaced rods as shown in Fig. P-236. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass M which can be supported.

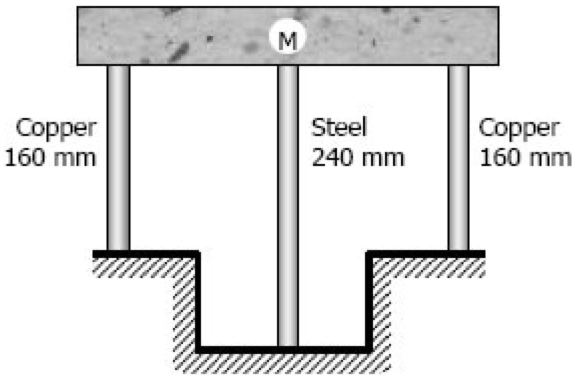
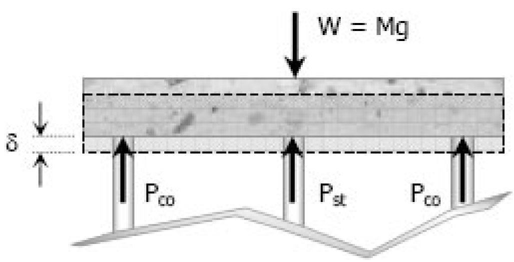


Figure P-236 and P-237

Solution 236



$$\delta_{co} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{co} L}{120\,000} = \frac{\sigma_{st} L}{200\,000}$$
$$10\sigma_{co} = 9\sigma_{st}$$

When σ_{st} = 140 MPa

$\sigma_{co} = \frac{9}{10}(140)$

$\sigma_{co} = 126 \text{ MPa } > 70 \text{ MPa}$**(not ok!)**

When σ_{co} = 70 MPa

$\sigma_{st} = \frac{10}{9}(70)$

$\sigma_{st} = 77.78 \text{ MPa } lt; 140 \text{ MPa}$**(ok!)**

Use **σ_{co} = 70 MPa** and **σ_{st} = 77.78 MPa**

$$\Sigma F_V = 0$$
$$2P_{co} + P_{st} = W$$
$$2(\sigma_{co} A_{co}) + \sigma_{st} A_{st} = Mg$$
$$2[\,70(900)\,] + 77.78(1200) = M(9.81)$$
$$M = 22358.4 \text{ kg}$$ →***answer***

Problem 237

In Problem 236, how should the lengths of the two identical copper rods be changed so that each material will be stressed to its allowable limit?

Solution 237

Use **σ_{co} = 70 MPa** and **σ_{st} = 140 MPa**

$$\delta_{co} = \delta_{st}$$

$$\left(\frac{\sigma L}{E}\right)_{co} = \left(\frac{\sigma L}{E}\right)_{st}$$

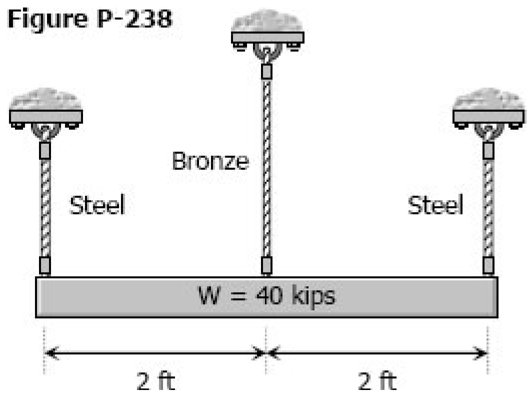
$$\frac{70L_{co}}{120\,000} = \frac{140(240)}{200\,000}$$

$$L_{co} = 288 \text{ mm}$$ ***answer***

Problem 238

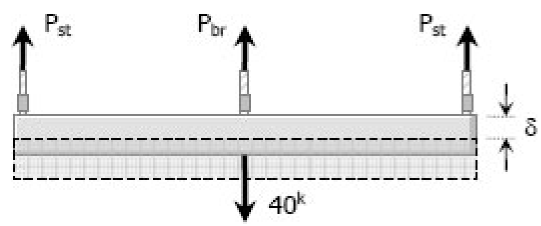
The lower ends of the three bars in Fig. P-238 are at the same level before the uniform rigid block weighing 40 kips is attached. Each steel bar has a length of 3 ft, and area of 1.0 in.^{2}, and E = 29 × 10^6 psi. For the bronze bar, the area is 1.5 in.^{2} and E = 12 × 10^6 psi. Determine (a) the length of the bronze bar so that the load on each steel bar is twice the load on the bronze bar, and (b) the length of the bronze that will make the steel stress twice the bronze stress.

Figure P-238



Solution 238

(a) Condition: $P_{st} = 2P_{br}$



$\Sigma F_V = 0$
$2P_{st} + P_{br} = 40$
$2(2P_{br}) + P_{br} = 40$
$P_{br} = 8\,\text{kips}$
$P_{st} = 2(8) = 16\,\text{kips}$

$\delta_{br} = \delta_{st}$
$\left(\dfrac{PL}{AE}\right)_{br} = \left(\dfrac{PL}{AE}\right)_{st}$
$\dfrac{8000\,L_{br}}{1.5(12\times 10^6)} = \dfrac{16000\,(3\times 12}{1.0(29\times 10^6)}$
$L_{br} = 44.69\,\text{in}$
$L_{br} = 3.72\,\text{ft}$ → **answer**

(b) Condition: $\sigma_{st} = 2\sigma_{br}$

$\Sigma F_V = 0$
$2P_{st} + P_{br} = 40$
$2(\sigma_{st}\,A_{st}) + \sigma_{br}\,A_{br} = 40$
$2\left[\,(2\sigma_{br})\,A_{st}\,\right] + \sigma_{br}\,A_{br} = 40$
$4\sigma_{br}(1.0) + \sigma_{br}(1.5) = 40$
$\sigma_{br} = 7.27\,\text{ksi}$
$\sigma_{st} = 2(7.27) = 14.54\,\text{ksi}$

$$\delta_{br} = \delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = \left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{7.27(1000)\ L_{br}}{12 \times 10^6} = \frac{14.54(1000)(3 \times 12)}{29 \times 10^6}$$
$$L_{br} = 29.79 \text{ in}$$
$$L_{br} = 2.48 \text{ ft}$$ →***answer***

Problem 239

The rigid platform in Fig. P-239 has negligible mass and rests on two steel bars, each 250.00 mm long. The center bar is aluminum and 249.90 mm long. Compute the stress in the aluminum bar after the center load P = 400 kN has been applied. For each steel bar, the area is 1200 mm2 and E = 200 GPa. For the aluminum bar, the area is 2400 mm2 and E = 70 GPa.

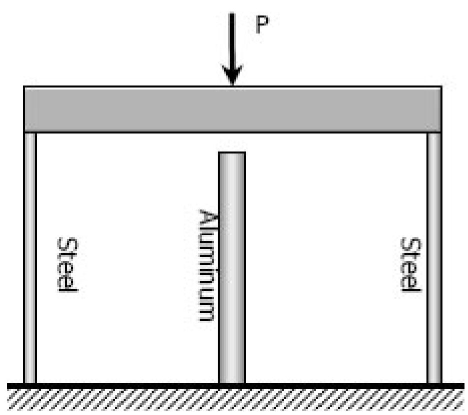
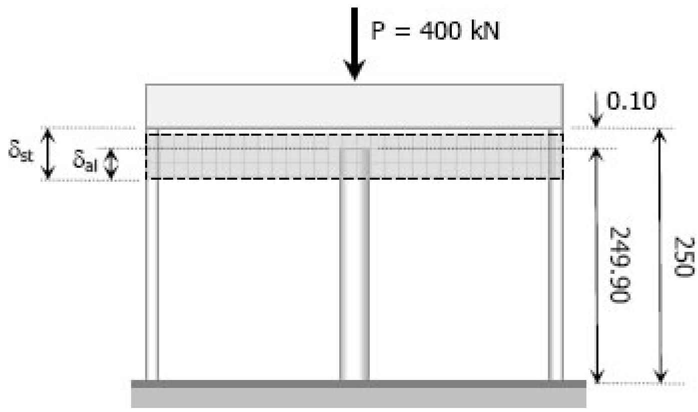
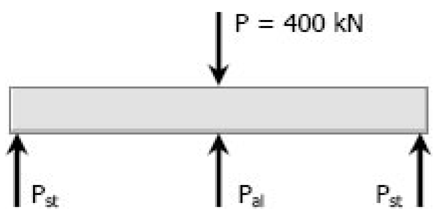


Figure P-239

Solution 239



$$\delta_{st} = \delta_{al} + 0.10$$
$$\left(\frac{\sigma\,L}{E}\right)_{st} = \left(\frac{\sigma\,L}{E}\right)_{al} + 0.10$$
$$\frac{\sigma_{st}(250)}{200\,000} = \frac{\sigma_{al}(249.90)}{70\,000} + 0.10$$
$$0.00125\sigma_{st} = 0.00357\sigma_{al} + 0.10$$
$$\sigma_{st} = 2.856\sigma_{al} + 80$$



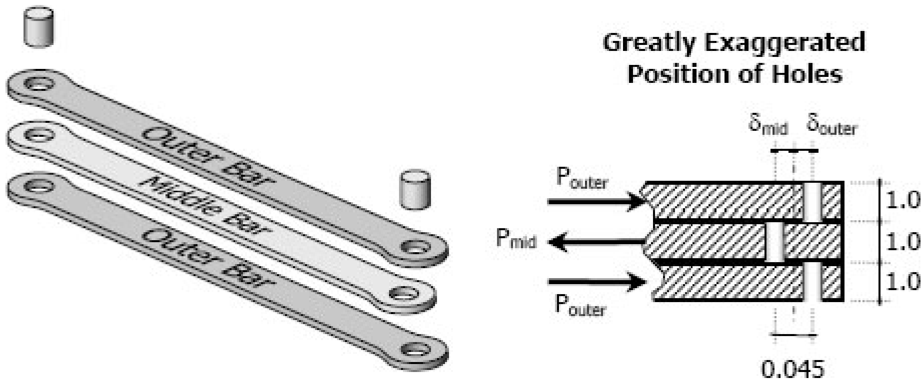
$$\Sigma F_V = 0$$
$$2P_{st} + P_{al} = 400\,000$$
$$2\sigma_{st}\,A_{st} + \sigma_{al}\,A_{al} = 400\,000$$
$$2(2.856\sigma_{al} + 80)1200 + \sigma_{al}(2400) = 400\,000$$
$$9254.4\sigma_{al} + 192\,000 = 400\,000$$
$$\sigma_{al} = 22.48\ \text{MPa}\quad \rightarrow$$ ***answer***

Problem 240

Three steel eye-bars, each 4 in. by 1 in. in section, are to be assembled by driving rigid 7/8-in.-diameter drift pins through holes drilled in the ends of the bars. The center-line spacing between the holes is 30 ft in the two outer bars, but 0.045 in. shorter in the middle bar. Find the shearing stress developed in the drip pins. Neglect local deformation at the holes.

Solution 240

Middle bar is 0.045 inch shorter between holes than outer bars.



$$\Sigma F_H = 0$$
$$P_{mid} = 2P_{outer}$$

$$\delta_{outer} + \delta_{mid} = 0.045$$
$$\left(\frac{PL}{AE}\right)_{outer} + \left(\frac{PL}{AE}\right)_{mid} = 0.045$$
$$\frac{P_{outer}(30 \times 12)}{[\,1.0(4.0)\,]E} + \frac{P_{mid}(30 \times 12 - 0.045)}{[\,1.0(4.0)\,]E} = 0.045$$
$$360P_{outer} + 359.955P_{mid} = 0.18E$$
$$360P_{outer} + 359.955(2P_{outer}) = 0.18E$$

(For steel: E = 29 × 10^6 psi)

$$1079.91P_{outer} = 0.18(29 \times 10^6)$$
$$P_{outer} = 4833.74\,\text{lb}$$

$$P_{mid} = 2(4833.74)$$
$$P_{mid} = 9667.48\,\text{lb}$$

Use shear force $V = P_{mid}$

Shearing stress of drip pins (double shear):

$$\tau = \frac{V}{A} = \frac{9667.48}{2\,[\,\frac{1}{4}\pi\,(\frac{7}{8})^2\,]}$$
$$\tau = 8038.54\ \text{psi}$$ → ***answer***

Problem 241

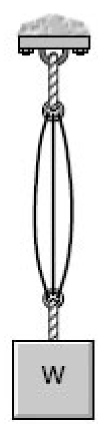


Figure P-241

As shown in Fig. P-241, three steel wires, each 0.05 in.2 in area, are used to lift a load W = 1500 lb. Their unstressed lengths are 74.98 ft, 74.99 ft, and 75.00 ft.
(a) What stress exists in the longest wire?
(b) Determine the stress in the shortest wire if W = 500 lb.

Solution 241

Let L_1 = 74.98 ft; L_2 = 74.99 ft; and L_3 = 75.00 ft

Part (a)

Bring L_1 and L_2 into L_3 = 75 ft length:
(For steel: E = 29 × 10^6 psi)

$$\delta = \frac{PL}{AE}$$

For L_1:

$$(75-74.98)(12)=\frac{P_1(74.98\times 12)}{0.05(29\times 10^6)}$$
$$P_1=386.77\,\mathrm{lb}$$

For L_2:

$$(75-74.99)(12)=\frac{P_2(74.99\times 12)}{0.05(29\times 10^6)}$$
$$P_2=193.36\,\mathrm{lb}$$

Let $P = P_3$ (Load carried by L_3)

$P + P_2$ (Total load carried by L_2)
$P + P_1$ (Total load carried by L_1)

$$\Sigma F_V=0$$
$$(P+P_1)+(P+P_2)+P=W$$
$$3P+386.77+193.36=1500$$
$$P=306.62lb=P_3$$

$$\sigma_3=\frac{P_3}{A}=\frac{306.62}{0.05}$$
$$\sigma_3=6132.47\ \mathrm{psi}\rightarrow$$ ***answer***

Part (b)

From the above solution:
$P_1 + P_2 = 580.13$ lb > 500 lb (L_3 carries no load)

Bring L_1 into L_2 = 74.99 ft

$$\delta = \frac{PL}{AE}$$
$$(74.99 - 74.98)(12) = \frac{P_1(74.98 \times 12)}{0.05(29 \times 10^6)}$$
$$P_1 = 193.38 \text{ lb}$$

Let P = P_2 (Load carried by L_2)

P + P_1 (Total load carried by L_1)

$$\Sigma F_V = 0$$
$$(P + P_1) + P = 500$$
$$2P + 193.38 = 500$$
$$P = 153.31 \text{ lb}$$

$$P + P_1 = 153.31 + 193.38$$
$$P + P_1 = 346.69 \text{ lb}$$

$$\sigma = \frac{P + P_1}{A} = \frac{346.69}{0.05}$$
$\sigma = 6933.8$ psi $\rightarrow$***answer***

Problem 242

The assembly in [Fig. P-242]() consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, Δ = 5 mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.

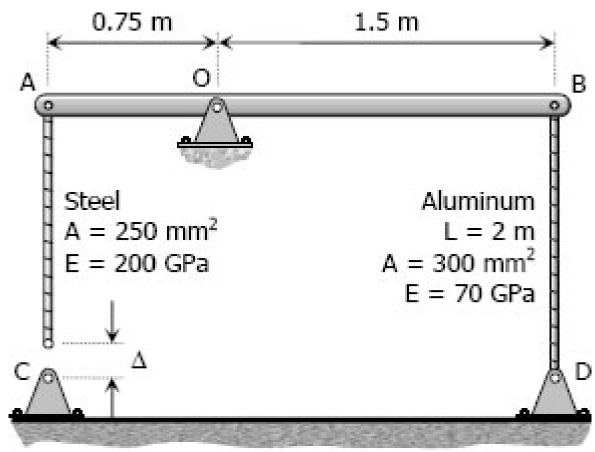
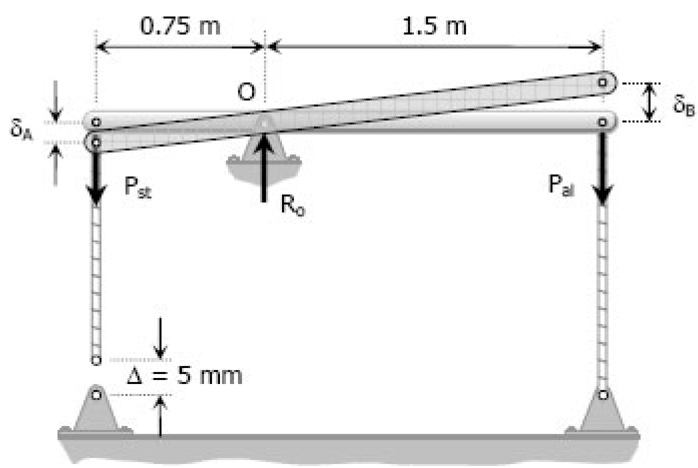


Figure P-242

Solution 242



$\Sigma M_O = 0$
$0.75P_{st} = 1.5P_{al}$
$P_{st} = 2P_{al}$
$\sigma_{st}A_{st} = 2(\sigma_{al}A_{al})$

$$\sigma_{st} = \frac{2\sigma_{al}\, A_{al}}{A_{st}}$$
$$\sigma_{st} = \frac{2\,[\,\sigma_{al}(3000\,]}{250}$$
$$\sigma_{st} = 2.4\sigma_{al}$$

$$\delta_{al} = \delta_B$$

By ratio and proportion:

$$\frac{\delta_A}{0.75} = \frac{\delta_B}{1.5}$$
$$\delta_A = 0.5\delta_B$$
$$\delta_A = 0.5\delta_{al}$$

$$\Delta = \delta_{st} + \delta_A$$
$$5 = \delta_{st} + 0.5\delta_{al}$$

$$5 = \frac{\sigma_{st}(2\,000 - 5)}{250(200\,000)} + 0.5\left[\frac{\sigma_{al}(2000)}{300(70\,000)}\right]$$

$$5 = (3.99 \times 10^{-5})\,\sigma_{st} + (4.76 \times 10^{-5})\,\sigma_{al}$$
$$\sigma_{al} = 105\,000 - 0.8379\,\sigma_{st}$$
$$\sigma_{al} = 105\,000 - 0.8379(2.4\,\sigma_{al})$$
$$3.01096\sigma_{al} = 105\,000$$
$$\sigma_{al} = 34\,872.6\ \text{MPa}$$ → ***answer***

Problem 243

A homogeneous rod of constant cross section is attached to unyielding supports. It carries an axial load P applied as shown in Fig. P-243. Prove that the reactions are given by $R_1 = Pb/L$ and $R_2 = Pa/L$.

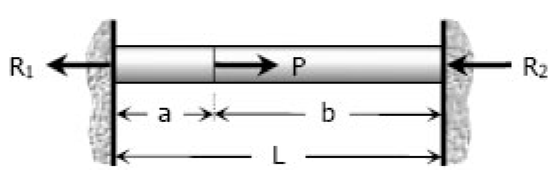
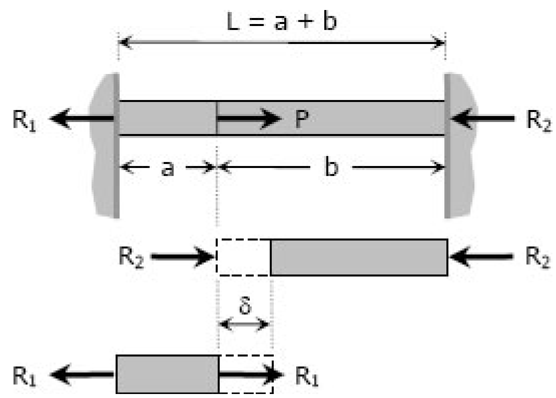


Figure P-243

Solution 243



$\Sigma F_H = 0$
$R_1 + R_2 = P$
$R_2 = P - R_1$

$\delta_1 = \delta_2 = \delta$
$\left(\dfrac{PL}{AE}\right)_1 = \left(\dfrac{PL}{AE}\right)_2$
$\dfrac{R_1\,a}{AE} = \dfrac{R_2\,b}{AE}$
$R_1\,a = R_2\,b$

$R_1\,a = (P - R_1)b$
$R_1\,a = Pb - R_1\,b$
$R_1\,(a + b) = Pb$
$R_1\,L = Pb$
$R_1 = Pb/L$ ***ok!***

$R_2 = P - Pb/L$
$R_2 = \dfrac{P\,(L - b)}{L}$
$R_2 = Pa/L$ ***ok!***

Problem 244

A homogeneous bar with a cross sectional area of 500 mm^2 is attached to rigid supports. It carries the axial loads P_1 = 25 kN and P_2 = 50 kN, applied as shown in Fig. P-244. Determine the stress in segment BC. (Hint: Use the results of Prob. 243, and compute the reactions caused by P_1 and P_2 acting separately. Then use the principle of superposition to compute the reactions when both loads are applied.)

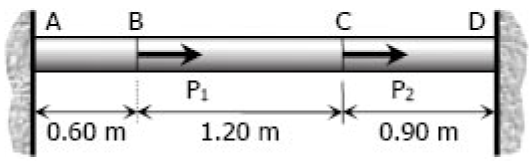
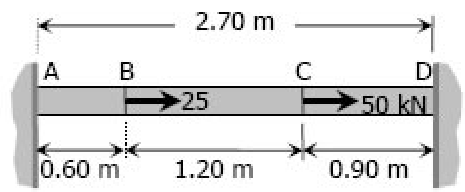


Figure P-244

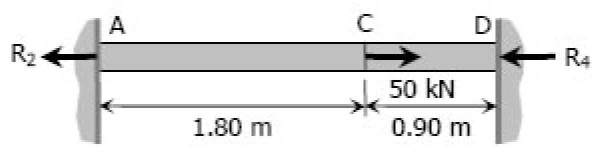
Solution 244



From the results of Solution to Problem 243:

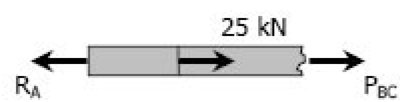
$R_1 = 25(2.10)/2.70$
$R_1 = 19.44\ \text{kN}$

$R_2 = 50(0.90)/2.70$
$R_2 = 16.67\ \text{kN}$



$R_A = R_1 + R_2$
$R_A = 19.44 + 16.67$
$R_A = 36.11\ \text{kN}$

For segment BC



$P_{BC} + 25 = R_A$
$P_{BC} + 25 = 36.11$
$P_{BC} = 11.11\ \text{kN}$

$$\sigma_{BC} = \frac{P_{BC}}{A} = \frac{11.11(1000)}{500}$$
$\sigma_{BC} = 22.22\ \text{MPa}$ $\rightarrow$ ***answer***

$$\sigma_{st} = 72.40(1000)/2000$$
$$\sigma_{st} = 36.20\text{ MPa} \rightarrow$$ ***answer***

$$\sigma_{br} = 162.40(1000)/1200$$
$$\sigma_{br} = 135.33\text{ MPa} \rightarrow$$ ***answer***

Problem 245

The composite bar in Fig. P-245 is firmly attached to unyielding supports. Compute the stress in each material caused by the application of the axial load P = 50 kips.

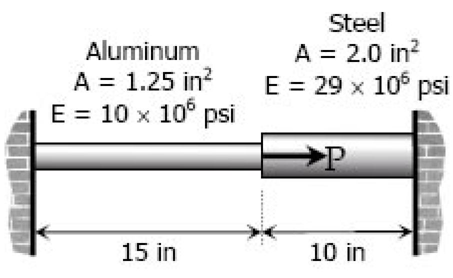
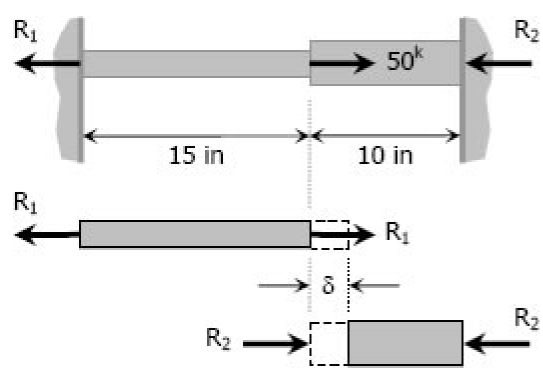


Figure P-245 and P-246

Solution 245



$$\Sigma F_H = 0$$
$$R_1 + R_2 = 50\,000$$
$$R_1 = 50\,000 - R_2$$

$$\delta_{al} = \delta_{st} = \delta$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st}$$
$$\frac{R_1\,(15)}{1.25(10 \times 10^6)} = \frac{R_2\,(10)}{2.0(29 \times 10^6)}$$
$$R_2 = 6.96R_1$$
$$R_2 = 6.96(50\,000 - R_2)$$
$$7.96R_2 = 348\,000$$
$$R_2 = 43\,718.59\text{ lb}$$

$$\sigma_{st} = \frac{R_2}{A_{st}} = \frac{43\,718.59}{2.0}$$
$$\sigma_{st} = 21\,859.30 \text{ psi}$$ →***answer***

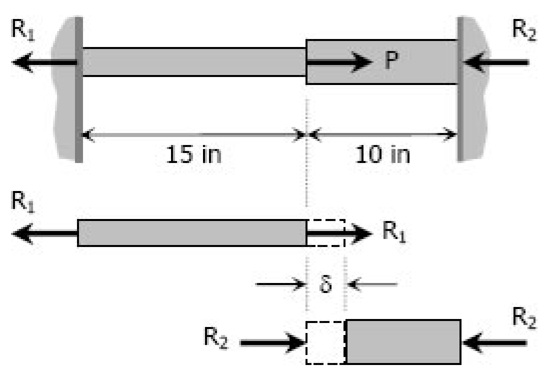
$$R_1 = 50\,000 - 43\,718.59$$
$$R_1 = 6281.41 \text{ lb}$$

$$\sigma_{al} = \frac{R_1}{A_{al}} = \frac{6281.41}{1.25}$$
$$\sigma_{al} = 5025.12 \text{ psi}$$ →***answer***

Problem 246

Referring to the composite bar in Problem 245, what maximum axial load P can be applied if the allowable stresses are 10 ksi for aluminum and 18 ksi for steel.

Solution 246



$$\delta_{st} = \delta_{al} = \delta$$
$$\left(\frac{\sigma\,L}{E}\right)_{st} = \left(\frac{\sigma\,L}{E}\right)_{al}$$
$$\frac{\sigma_{st}\,(10)}{29 \times 10^6} = \frac{\sigma_{al}\,(15)}{10 \times 10^6}$$
$$\sigma_{st} = 4.35\,\sigma_{al}$$

When σ$_{al}$ = 10 ksi

$$\sigma_{st} = 4.35(10)$$
$$\sigma_{st} = 43.5 \text{ ksi} > 18 \text{ ksi}$$ (***not ok!***)

When σ$_{st}$ = 18 ksi

$$18 = 4.35\,\sigma_{al}$$
$$\sigma_{al} = 4.14 \text{ ksi } lt; 10 \text{ ksi}$$ (***ok!***)

Use σ$_{al}$ = 4.14 ksi and σ$_{st}$ = 18 ksi

$$\Sigma F_H = 0$$
$$P = R_1 + R_2$$
$$P = \sigma_{al}\,A_{al} + \sigma_{st}\,A_{st}$$
$$P = 4.14(1.25) + 18(2.0)$$
$$P = 41.17 \text{ kips} \quad \rightarrow$$

Problem 247

The composite bar in Fig. P-247 is stress-free before the axial loads P_1 and P_2 are applied. Assuming that the walls are rigid, calculate the stress in each material if P_1 = 150 kN and P_2 = 90 kN.

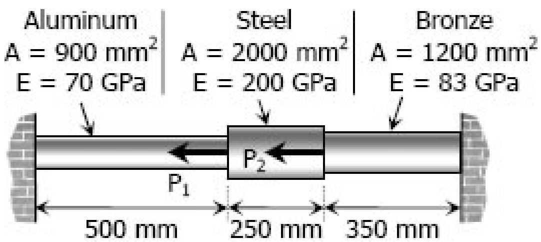
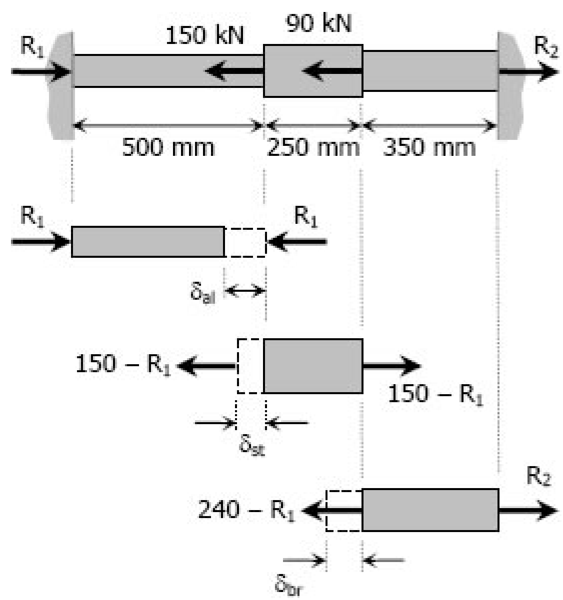


Figure P-247 and P-248

Solution 247



From the FBD of each material shown:
δ_{al} is shortening
δ_{st} and δ_{br} are lengthening

$R_2 = 240 - R_1$
$P_{al} = R_1$
$P_{st} = 150 - R_1$
$P_{br} = R_2 = 240 - R_1$

$\delta_{al} = \delta_{st} + \delta_{br}$

$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br}$$

$$\frac{R_1\,(500)}{900(70\,000)} = \frac{(150 - R_1)(250)}{2000(200\,000)} + \frac{(240 - R_1)(35}{1200(83\,000)}$$

$$\frac{R_1}{126\,000} = \frac{150 - R_1}{1\,600\,000} + \frac{(240 - R_1)7}{1\,992\,000}$$

$\frac{1}{63}R_1 = \frac{1}{800}(150 - R_1) + \frac{7}{996}(240 - R_1)$

$(\frac{1}{63} + \frac{1}{800} + \frac{7}{996})R_1 = \frac{1}{800}(150) + \frac{7}{996}(240)$

$R_1 = 77.60\ \text{kN}$

$P_{al} = R_1 = 77.60\,\text{kN}$
$P_{st} = 150 - 77.60 = 72.40\,\text{kN}$
$P_{br} = 240 - 77.60 = 162.40\,\text{kN}$

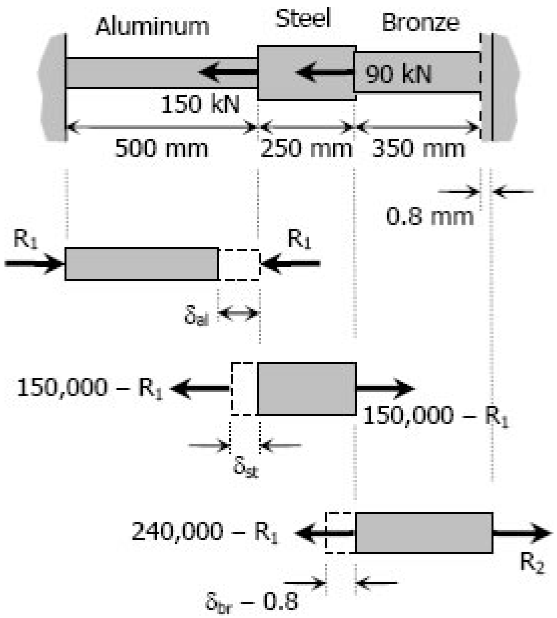
$\sigma = P/A$

$$\sigma_{al} = 77.60(1000)/900$$
$$\sigma_{al} = 86.22 \text{ MPa} \rightarrow \textbf{\textit{answer}}$$

Problem 248

Solve Problem 247 if the right wall yields 0.80 mm.

Solution 248



$$\delta_{al} = \delta_{st} + (\delta_{br} + 0.8)$$
$$\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{br} + 0.8$$
$$\frac{R_1\,(500)}{900(70\,000)} = \frac{(150\,000 - R_1)(250)}{2000(200\,000)} + \frac{(240\,000 - R_1)(350)}{1200(83\,000)} + 0.8$$
$$\frac{R_1}{126\,000} = \frac{150\,000 - R_1}{1\,600\,000} + \frac{7(240\,000 - R_1)}{1\,992\,000} + 0.8$$
$$\tfrac{1}{63}R_1 = \tfrac{1}{800}(150\,000 - R_1) + \tfrac{7}{996}(240\,000 - R_1) + 1600$$
$$(\tfrac{1}{63} + \tfrac{1}{800} + \tfrac{7}{996})R_1 = \tfrac{1}{800}(150\,000) + \tfrac{7}{996}(240\,000) + 1600$$
$$R_1 = 143\,854 \text{ N} = 143.854 \text{ kN}$$

$P_{al} = R_1 = 143.854\,\text{kN}$
$P_{st} = 150 - R_1 = 150 - 143.854 = 6.146\,\text{kN}$
$P_{br} = R_2 = 240 - R_1 = 240 - 143.854 = 96.146\,\text{kN}$

$\sigma = P/A$

$\sigma_{al} = 143.854(1000)/900$
$\sigma_{al} = 159.84\,\text{MPa}$ →***answer***

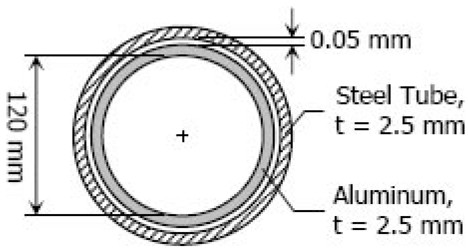
$\sigma_{st} = 6.146(1000)/2000$
$\sigma_{st} = 3.073\,\text{MPa}$ →***answer***

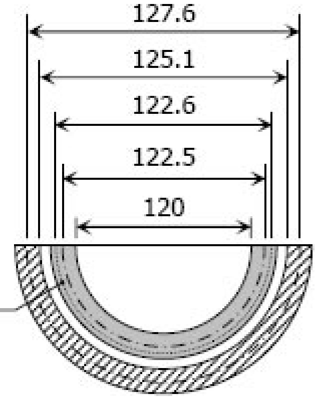
$\sigma_{br} = 96.146(1000)/1200$
$\sigma_{br} = 80.122\,\text{MPa}$ →***answer***

Problem 249

There is a radial clearance of 0.05 mm when a steel tube is placed over an aluminum tube. The inside diameter of the aluminum tube is 120 mm, and the wall thickness of each tube is 2.5 mm. Compute the contact pressure and tangential stress in each tube when the aluminum tube is subjected to an internal pressure of 5.0 MPa.

Solution 249





Internal pressure of aluminum tube to cause contact with the steel:

$$\delta_{al} = \left(\frac{\sigma\, L}{E}\right)_{al}$$

$$\pi(122.6 - 122.5) = \frac{\sigma_1\,(122.5\pi)}{70\,000}$$

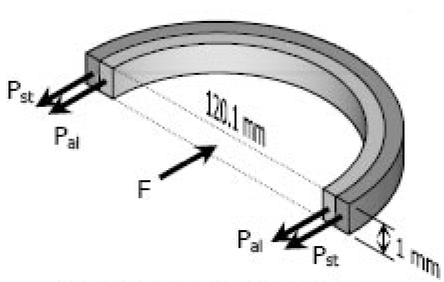
$$\sigma_1 = 57.143\,\text{MPa}$$

$$\frac{p_1\, D}{2t} = 57.143$$

$$\frac{p_1\,(120)}{2(2.5)} = 57.143$$

$p_1 = 2.381\ \text{MPa}$ → pressure that causes aluminum to contact with the steel, further increase of pressure will expand both aluminum and steel tubes.

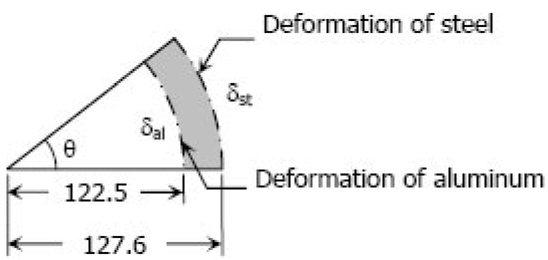
Let p_c = contact pressure between steel and aluminum tubes



FBD for p ≥ 2.381 MPa

$2P_{st} + 2P_{al} = F$
$2P_{st} + 2P_{al} = 5.0(120.1)(1)$
$P_{st} + P_{al} = 300.25$ →Equation (1)

The relationship of deformations is (from the figure):



Geometric relation of deformations

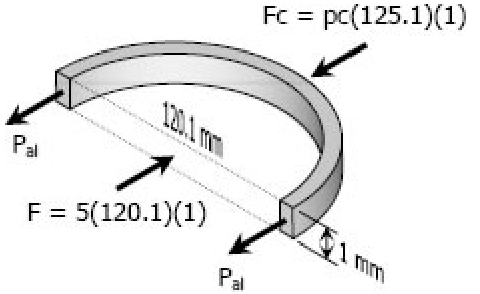
$\delta_{st} = 127.6\theta$
$\theta = \delta_{st}/127.6$

$\delta_{al} = 122.5\theta$
$\delta_{al} = 122.5(\delta_{st}/127.6)$
$\delta_{al} = 0.96\,\delta_{st}$
$\left(\dfrac{PL}{AE}\right)_{al} = 0.96\,\left(\dfrac{PL}{AE}\right)_{st}$
$\dfrac{P_{al}\,(122.5\pi)}{2.5(70\,000)} = 0.96\,\left[\dfrac{P_{st}\,(127.6)}{2.5(200\,000)}\right]$
$P_{al} = 0.35P_{st}$ →Equation (2)

From Equation (1)
$P_{st} + 0.35P_{st} = 300.25$
$P_{st} = 222.41\,\mathrm{N}$

$P_{al} = 0.35(222.41)$
$P_{al} = 77.84\,\mathrm{N}$

Contact Force



$$F_c + 2P_{st} = F$$
$$p_c(125.1)(1) + 2(77.84) = 5(120.1)(1)$$
$$p_c = 3.56 \text{ MPa}$$ →***answer***

Problem 250

In the assembly of the bronze tube and steel bolt shown in Fig. P-250, the pitch of the bolt thread is p = 1/32 in.; the cross-sectional area of the bronze tube is 1.5 in.^{2} and of steel bolt is 3/4 in.^{2} The nut is turned until there is a compressive stress of 4000 psi in the bronze tube. Find the stresses if the nut is given one additional turn. How many turns of the nut will reduce these stresses to zero? Use E_{br} = 12 × 10^6 psi and E_{st} = 29 × 10^6 psi.

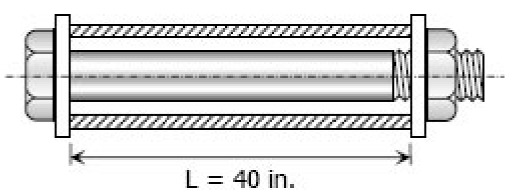
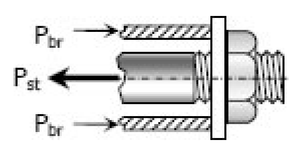


Figure P-250

Solution 250



$$P_{st} = P_{br}$$
$$A_{st}\,\sigma_{st} = P_{br}\,\sigma_{br}$$

$\frac{3}{4}\sigma_{st} = 1.5\,\sigma_{br}$
$\sigma_{st} = 2\sigma_{br}$

For one turn of the nut:
$\delta_{st} + \delta_{br} = \frac{1}{32}$
$\left(\dfrac{\sigma\,L}{E}\right)_{st} + \left(\dfrac{\sigma\,L}{E}\right)_{br} = \dfrac{1}{32}$
$\dfrac{\sigma_{st}(40)}{29\times10^6} + \dfrac{\sigma_{br}(40)}{12\times10^6} = \dfrac{1}{32}$
$\sigma_{st} + \frac{29}{12}\sigma_{br} = 22\,656.25$
$2\sigma_{br} + \frac{29}{12}\sigma_{br} = 22\,656.25$
$\sigma_{br} = 5129.72\text{ psi}$
$\sigma_{st} = 2(5129.72) = 10259.43\text{ psi}$

Initial stresses:
$\sigma_{br} = 4000\text{ psi}$
$\sigma_{st} = 2(4000) = 8000\text{ psi}$

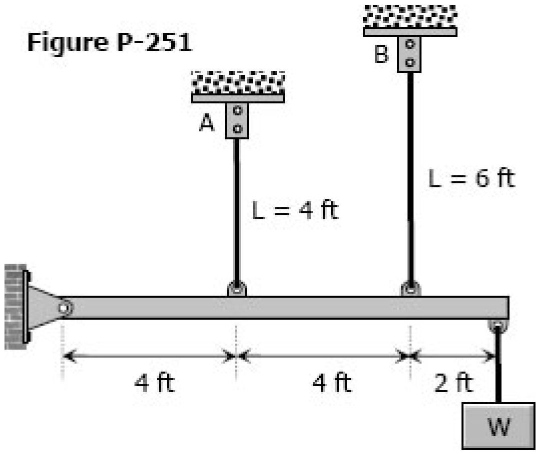
Final stresses:
$\sigma_{br} = 4000 + 5129.72 = 9\,129.72 \;\;\text{psi}$ → ***answer***
$\sigma_{st} = 2(9129.72) = 18\,259.4 \;\;\text{psi}$ → ***answer***

Required number of turns to reduce σ_{br} to zero:
$n = \dfrac{9129.72}{5129.72} = 1.78 \;\;\text{turns}$

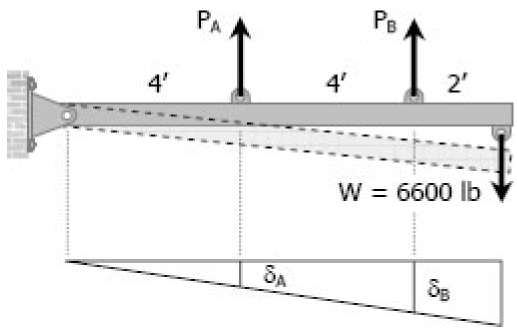
The nut must be turned back by **1.78 turns**

Problem 251

The two vertical rods attached to the light rigid bar in Fig. P-251 are identical except for length. Before the load W was attached, the bar was horizontal and the rods were stress-free. Determine the load in each rod if W = 6600 lb.



Solution 251



$\Sigma M_{pin\ support} = 0$
$4P_A + 8P_B = 10(6600)$
$P_A + 2P_B = 16500$ →equation (1)

By ratio and proportion
$\dfrac{\delta_A}{4} = \dfrac{\delta_B}{8}$
$\delta_A = 0.5\delta_B$
$\left(\dfrac{PL}{AE}\right)_A = 0.5\left(\dfrac{PL}{AE}\right)_B$
$\dfrac{P_A(4)}{AE} = \dfrac{0.5P_B(6)}{AE}$
$P_A = 0.75P_B$

From equation (1)
$0.75P_B + 2P_B = 16500$
$P_B = 6000\ \text{lb}$ →***answer***

$P_A = 0.75(6000)$
$P_A = 4500\ \text{lb}$ →***answer***

Problem 253

As shown in Fig. P-253, a rigid beam with negligible weight is pinned at one end and attached to two vertical rods. The beam was initially horizontal before the load W = 50 kips was applied. Find the vertical movement of W.

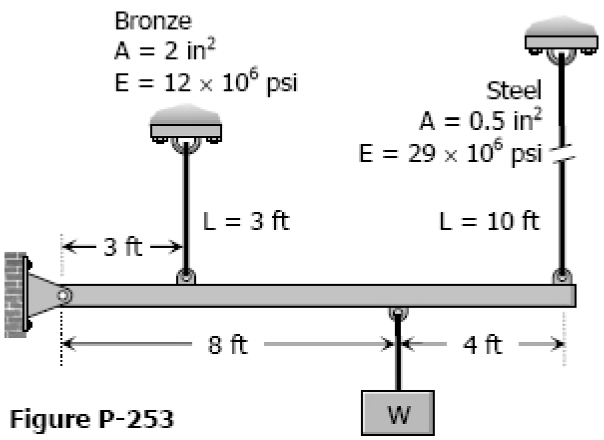
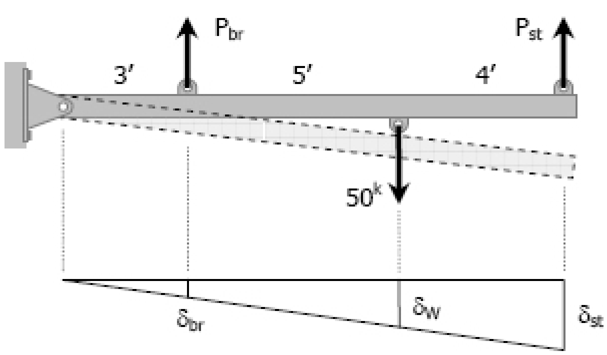


Figure P-253

Solution 253

$$\Sigma M_{pin\ support} = 0$$
$$3P_{br} + 12P_{st} = 8(50\,000)$$
$$3P_{br} + 12P_{st} = 400\,000 \quad \rightarrow \text{Equation (1)}$$



$$\frac{\delta_{st}}{12} = \frac{\delta_{br}}{3}$$
$$\delta_{st} = 4\delta_{br}$$
$$\left(\frac{PL}{AE}\right)_{st} = 4\left(\frac{PL}{AE}\right)_{br}$$
$$\frac{P_{st}(10)}{0.5(29 \times 10^6)} = 4\left[\frac{P_{br}(3)}{2(12 \times 10^6)}\right]$$
$$P_{st} = 0.725P_{br}$$

From equation (1)

$$3P_{br} + 12(0.725P_{br}) = 400\,000$$
$$P_{br} = 34\,188.03\ \text{lb}$$

$$\delta_{br} = \left(\frac{PL}{AE}\right)_{br} = \frac{34\,188.03(3 \times 12)}{2(12 \times 10^6)}$$
$$\delta_{br} = 0.0513\ \text{in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{br}}{3}$$
$$\delta_W = \tfrac{8}{3}\delta_{br}$$
$$\delta_W = \tfrac{8}{3}(0.0513)$$
$$\delta_W = 0.1368\ \text{in}$$ →***answer***

Check by δ_{st}:

$$P_{st} = 0.725P_{br} = 0.725(34\,188.03)$$
$$P_{st} = 24\,786.32\ \text{lb}$$

$$\delta_{st} = \left(\frac{PL}{AE}\right)_{st} = \frac{24\,786.32(10 \times 12)}{0.5(29 \times 10^6)}$$
$$\delta_{br} = 0.2051\ \text{in}$$

$$\frac{\delta_W}{8} = \frac{\delta_{st}}{12}$$
$$\delta_W = \tfrac{2}{3}\delta_{st}$$

$\delta_W = \frac{2}{3}(0.2051)$
$\delta_W = 0.1368 \text{ in}$ →***ok!***

Problem 254

As shown in Fig. P-254, a rigid bar with negligible mass is pinned at O and attached to two vertical rods. Assuming that the rods were initially stress-free, what maximum load P can be applied without exceeding stresses of 150 MPa in the steel rod and 70 MPa in the bronze rod.

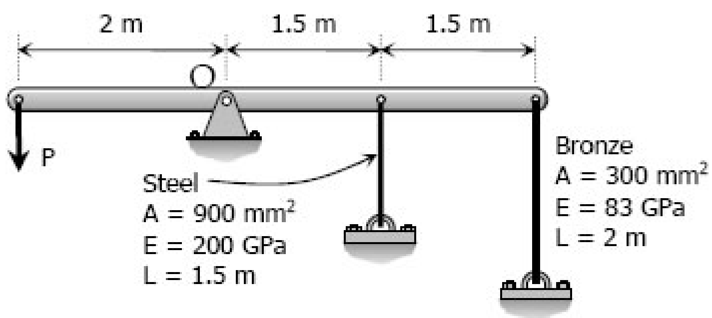
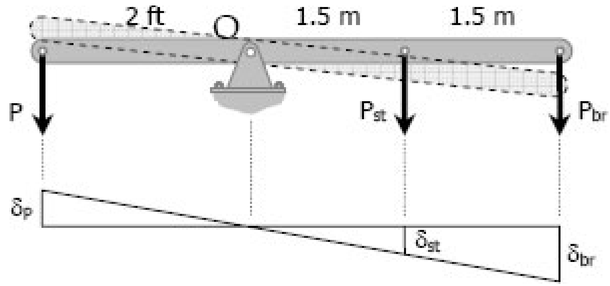


Figure P-254

Solution 254

$\Sigma M_O = 0$
$2P = 1.5P_{st} + 3P_{br}$
$2P = 1.5(\sigma_{st}\, A_{st}) + 3(\sigma_{br}\, A_{br})$
$2P = 1.5\,[\,\sigma_{st}\,(900)\,] + 3\,[\,\sigma_{br}\,(300)\,]$
$2P = 1350\sigma_{st} + 900\sigma_{br}$
$P = 675\sigma_{st} + 450\sigma_{br}$



$$\frac{\delta_{br}}{3} = \frac{\delta_{st}}{1.5}$$
$$\delta_{br} = 2\delta_{st}$$
$$\left(\frac{\sigma L}{E}\right)_{br} = 2\left(\frac{\sigma L}{E}\right)_{st}$$
$$\frac{\sigma_{br}(2)}{83} = 2\left[\frac{\sigma_{st}(1.5)}{200}\right]$$
$$\sigma_{br} = 0.6225\sigma_{st}$$

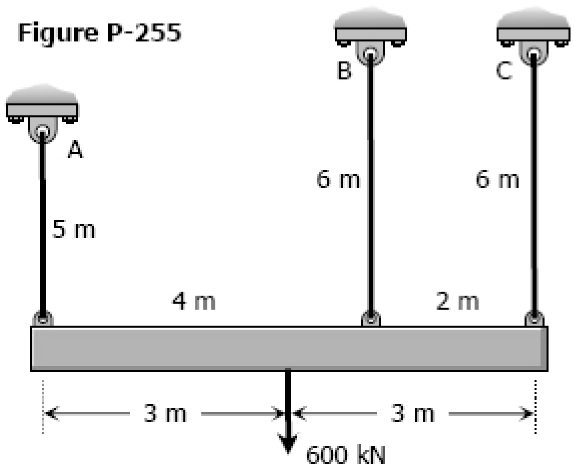
When $\sigma_{st} = 150\,\text{MPa}$
$$\sigma_{br} = 0.6225(150)$$
$\sigma_{br} = 93.375\,\text{MPa} > 70\,\text{MPa}$(*not ok!*)

When $\sigma_{br} = 70\,\text{MPa}$
$$70 = 0.6225\sigma_{st}$$
$\sigma_{st} = 112.45\,\text{MPa}lt; 150\,\text{MPa}$(*ok!*)

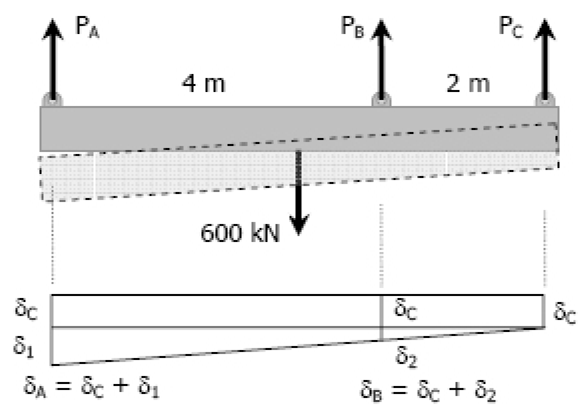
Use $\sigma_{st} = 112.45\,\text{MPa}$and $\sigma_{br} = 70\,\text{MPa}$
$$P = 675\sigma_{st} + 450\sigma_{br}$$
$$P = 675(112.45) + 450(70)$$
$$P = 107\,403.75\,\text{N}$$
$P = 107.4\,\text{kN}$ →***answer***

Problem 255

Shown in Fig. P-255 is a section through a balcony. The total uniform load of 600 kN is supported by three rods of the same area and material. Compute the load in each rod. Assume the floor to be rigid, but note that it does not necessarily remain horizontal.



Solution 255



$$\delta_B = \delta_C + \delta_2$$
$$\delta_2 = \delta_B - \delta_C$$
$$\frac{\delta_1}{6} = \frac{\delta_2}{2}$$
$$\delta_1 = 3\delta_2$$

$$\delta_A = \delta_C + \delta_1$$
$$\delta_A = \delta_C + 3\delta_2$$
$$\delta_A = \delta_C + 3(\delta_B - \delta_C)$$
$$\delta_A = 3\delta_B - 2\delta_C$$
$$\left(\frac{PL}{AE}\right)_A = 3\left(\frac{PL}{AE}\right)_B - 2\left(\frac{PL}{AE}\right)_C$$
$$\frac{P_A(5)}{AE} = \frac{3P_B(6)}{AE} - \frac{2P_C(6)}{AE}$$
$$P_A = 3.6P_B - 2.4P_C \quad \rightarrow \text{Equation (1)}$$

$\Sigma F_V = 0$
$P_A + P_B + P_C = 600$
$(3.6P_B - 2.4P_C) + P_B + P_C = 600$
$4.6P_B - 1.4P_C = 600$ →Equation (2)

$\Sigma M_A = 0$
$4P_B + 6P_C = 3(600)$
$P_B = 450 - 1.5P_C$ →Equation (3)

Substitute P_B = 450 - 1.5 P_C to Equation (2)
$4.6(450 - 1.5P_C) - 1.4P_C = 600$
$8.3P_C = 1470$
$P_C = 177.11\ \text{kN}$ →***answer***

From Equation (3)
$P_B = 450 - 1.5(177.11)$
$P_B = 184.34\ \text{kN}$ →***answer***

From Equation (1)
$P_A = 3.6(184.34) - 2.4(177.11)$
$P_A = 238.56\ \text{kN}$ →***answer***

Problem 256

Three rods, each of area 250 mm^2, jointly support a 7.5 kN load, as shown in Fig. P-256. Assuming that there was no slack or stress in the rods before the load was applied, find the stress in each rod. Use E_{st} = 200 GPa and E_{br} = 83 GPa.

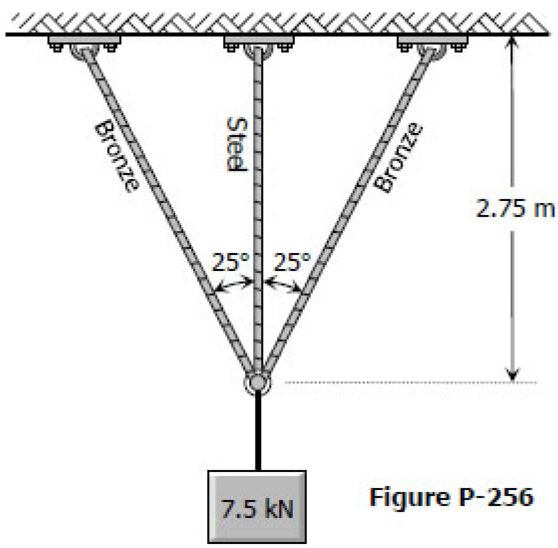
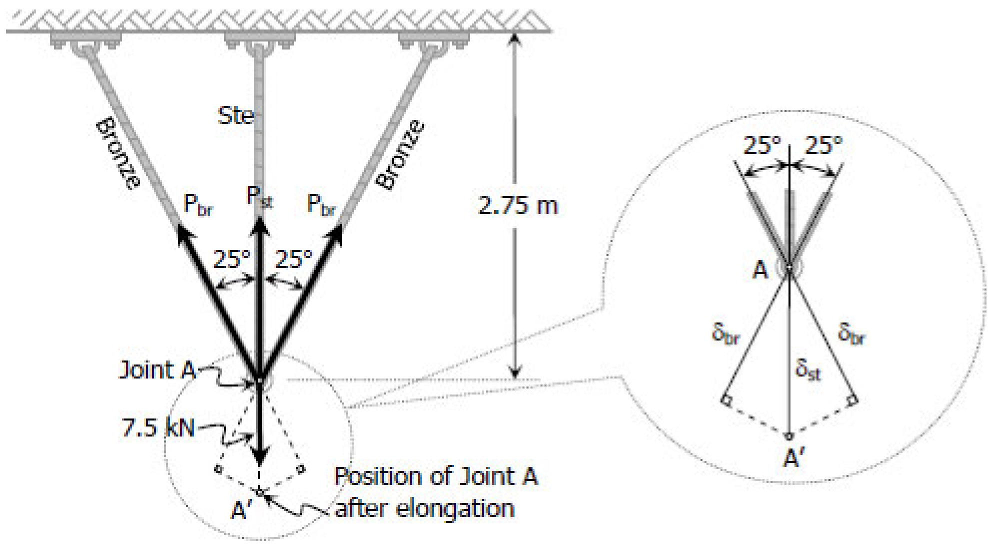


Figure P-256

Solution 256

$$\cos 25° = \frac{2.75}{L_{br}}$$
$$L_{br} = 3.03\ \text{m}$$



$\Sigma F_V = 0$
$2P_{br}\cos 25° + P_{st} = 7.5(1000)$
$P_{st} = 7500 - 1.8126P_{br}$
$\sigma_{st}\,A_{st} = 7500 - 1.8126\sigma_{br}\,A_{br}$
$\sigma_{st}(250) = 7500 - 1.8126\,[\,\sigma_{br}(250)\,]$
$\sigma_{st} = 30 - 1.8126\sigma_{br}$ →**Equation (1)**

$\cos 25° = \dfrac{\delta_{br}}{\delta_{st}}$
$\delta_{br} = 0.9063\delta_{st}$
$\left(\dfrac{\sigma L}{E}\right)_{br} = 0.9063\left(\dfrac{\sigma L}{E}\right)_{st}$
$\dfrac{\sigma_{br}(3.03)}{83} = 0.0963\left[\dfrac{\sigma_{st}(2.75)}{200}\right]$
$\sigma_{br} = 0.3414\sigma_{st}$ →**Equation (2)**

From Equation (1)
$\sigma_{st} = 30 - 1.8126(0.3414\sigma_{st})$
$\sigma_{st} = 18.53\ \text{MPa}$ →***answer***

From Equation (2)
$\sigma_{br} = 0.3414(18.53)$
$\sigma_{br} = 6.33\ \text{MPa}$ →***answer***

Problem 257

Three bars AB, AC, and AD are pinned together as shown in Fig. P-257. Initially, the assembly is stress free. Horizontal movement of the joint at A is prevented by a short horizontal strut AE. Calculate the stress in each bar and the force in the strut AE when the assembly is used to support the load W = 10 kips. For each steel bar, A = 0.3 in.^{2} and E = 29 × 10^6 psi. For the aluminum bar, A = 0.6 in.^{2} and E = 10 × 10^6 psi.

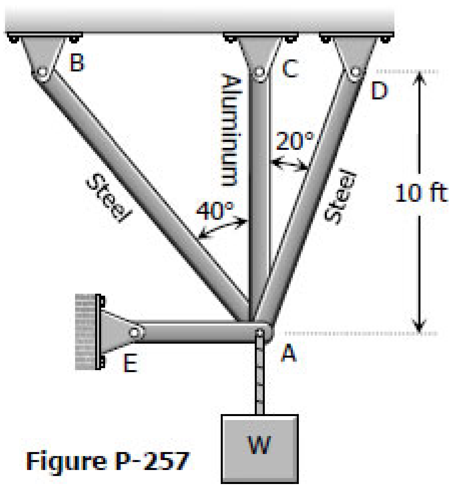


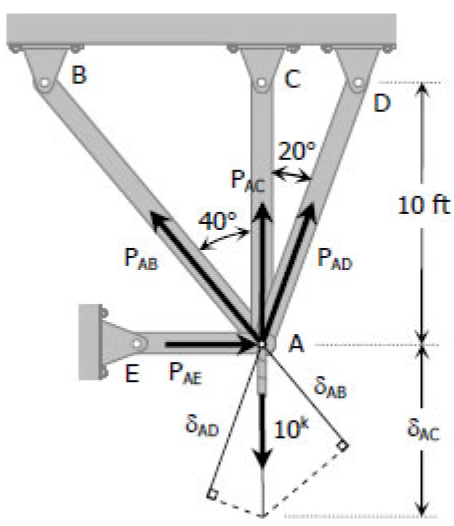
Figure P-257

Solution 257

$$\cos 40° = \frac{10}{L_{AB}}; \quad L_{AB} = 13.05 \text{ ft}$$

$$\cos 20° = \frac{10}{L_{AD}}; \quad L_{AD} = 10.64 \text{ ft}$$

$$\Sigma F_V = 0$$
$$P_{AB}\cos 40° + P_{AC} + P_{AD}\cos 20° = 10(1000)$$
$$0.7660 P_{AB} + P_{AC} + 0.9397 P_{AD} = 10000$$ → **Equation (1)**



$$\delta_{AB} = \cos 40° \, \delta_{AC} = 0.7660 \delta_{AC}$$
$$\left(\frac{PL}{AE}\right)_{AB} = 0.7660 \left(\frac{PL}{AE}\right)_{AC}$$
$$\frac{P_{AB}(13.05)}{0.3(29 \times 10^6)} = 0.7660 \left[\frac{P_{AC}(10)}{0.6(10 \times 10^6)}\right]$$
$$P_{AB} = 0.8511 P_{AC}$$ → **Equation (2)**

$$\delta_{AD} = \cos 20° \delta_{AC} = 0.9397\delta_{AC}$$
$$\left(\frac{PL}{AE}\right)_{AD} = 0.9397\left(\frac{PL}{AE}\right)_{AC}$$
$$\frac{P_{AB}(10.64)}{0.3(29 \times 10^6)} = 0.9397\left[\frac{P_{AC}(10)}{0.6(10 \times 10^6)}\right]$$
$P_{AD} = 1.2806P_{AC}$ →**Equation (3)**

Substitute P_{AB} of Equation (2) and P_{AD} of Equation (3) to Equation (1)

$$0.7660(0.8511P_{AC}) + P_{AC} + 0.9397(1.2806P_{AC}) = 10000$$
$$2.8553P_{AC} = 10\,000$$
$$P_{AC} = 3\,502.23\,textlb$$

From Equation (2)

$$P_{AB} = 0.8511(3\,502.23)$$
$$P_{AB} = 2\,980.75 \text{ lb}$$

From Equation (3)

$$P_{AD} = 1.2806(3\,502.23)$$
$$P_{AD} = 4\,484.96 \text{ lb}$$

Stresses:

$\sigma = P/A$

$\sigma_{AB} = 2980.75/0.3 = 9\,935.83 \text{ psi}$ →***answer***

$\sigma_{AC} = 3502.23/0.6 = 5\,837.05 \text{ psi}$ →***answer***

$\sigma_{AD} = 4484.96/0.3 = 14\,949.87 \text{ psi}$ →***answer***

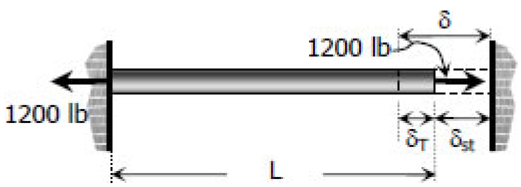
$$\Sigma F_H = 0$$
$$P_{AE} + P_{AD}\sin 20° = P_{AB}\sin 40°$$
$$P_{AE} = 2\,980.75\sin 40° - 4\,484.96\sin 20°$$
$P_{AE} = 382.04 \text{ lb}$ →***answer***

Problem 261

A steel rod with a cross-sectional area of 0.25 in^2 is stretched between two fixed points. The tensile load at 70°F is 1200 lb. What will be the stress at 0°F? At what temperature will the stress be zero? Assume α = 6.5 × 10^{-6} in/(in·°F) and E = 29 × 10^6 psi.

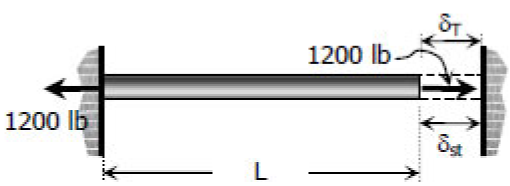
Solution 261

For the stress at 0°C:



$$\delta = \delta_T + \delta_{st}$$
$$\frac{\sigma\,L}{E} = \alpha\,L\,(\Delta T) + \frac{PL}{AE}$$
$$\sigma = \alpha\,E\,(\Delta T) + \frac{P}{E}$$
$$\sigma = (6.5 \times 10^{-6})(29 \times 10^{6})(70) + \frac{1200}{0.25}$$
$$\sigma = 17\,995\,\text{psi} = 18\ \text{ksi}$$ →***answer***

For the temperature that causes zero stress:

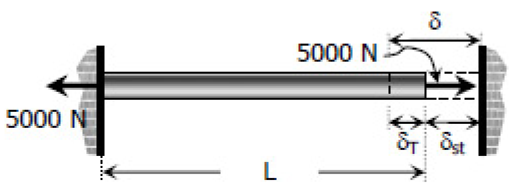


$$\delta_T = \delta_{st}$$
$$\alpha L\,(\Delta T) = \frac{PL}{AE}$$
$$\alpha\,(\Delta T) = \frac{P}{AE}$$
$$(6.5 \times 10^{-6})(T - 70) = \frac{1200}{0.25(29 \times 10^6)}$$
$T = 95.46\,°C$ →***answer***

Problem 262

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 262



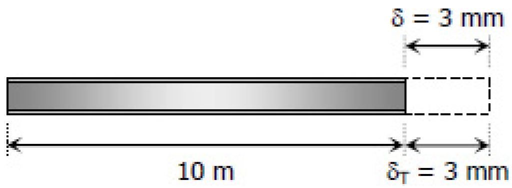
$$\delta = \delta_T + \delta_{st}$$
$$\frac{\sigma\,L}{E} = \alpha\,L\,(\Delta T) + \frac{PL}{AE}$$
$$\sigma = \alpha\,E\,(\Delta T) + \frac{P}{E}$$
$$130 = (11.7 \times 10^{-6})(200\,000)(40) + \frac{5000}{A}$$
$$A = \frac{5000}{36.4}137.36$$

$$\tfrac{1}{4}\pi d^2 = 137.36$$
d$= 13.22\ \text{ mm}$ → ***answer***

Problem 263

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 μm/(m·°C) and E = 200 GPa.

Solution 263



Temperature at which $\delta_T = 3\,\mathrm{mm}$**:**

$$\delta_T = \alpha\, L\, (\Delta T)$$
$$\delta_T = \alpha\, L\, (T_f\ -\ T_i)$$
$$3 = (11.7 \times 10^{-6})(10\,000)(T_f\ -\ 15)$$
$$T_f = 40.64^\circ\mathrm{C}\ \rightarrow \textit{\textbf{answer}}$$

Required stress:

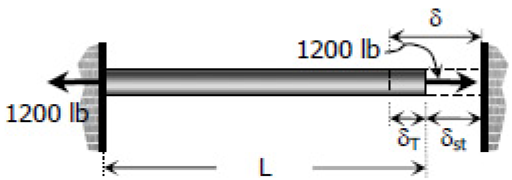
$$\delta = \delta_T$$
$$\frac{\sigma\, L}{E} = \alpha\, L\, (\Delta T)$$
$$\sigma = \alpha\, E\, (T_f\ -\ T_i)$$
$$\sigma = (11.7 \times 10^{-6})(200\,000)(40.64 - 15)$$
$$\sigma = 60\ \ \mathrm{MPa}\ \ \rightarrow \textit{\textbf{answer}}$$

Problem 264

A steel rod 3 feet long with a cross-sectional area of 0.25 in.^{2} is stretched between two fixed points. The tensile force is 1200 lb at 40°F. Using E = 29 × 10^6 psi and α = 6.5 Ã— 10^{-6} in./(in.·°F), calculate (a) the temperature at which the stress in the bar will be 10 ksi; and (b) the temperature at which the stress will be zero.

Solution 264

(a) Without temperature change:



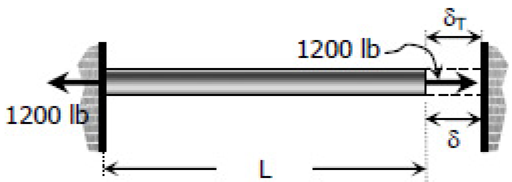
$$\sigma = \frac{P}{A} = \frac{1200}{0.25} = 4800 \text{ psi}$$
$$\sigma = 4.8\,\text{ksi}lt;10\,\text{ksi}$$

A drop of temperature is needed to increase the stress to 10 ksi. See figure above.

$$\delta = \delta_T + \delta_{st}$$
$$\frac{\sigma\,L}{E} = \alpha\,L\,(\Delta T) + \frac{PL}{AE}$$
$$\sigma = \alpha\,E\,(\Delta T) + \frac{P}{A}$$
$$10\,000 = (6.5 \times 10^{-6})(29 \times 10^6)(\Delta T) + \frac{1200}{0.25}$$
$$\Delta T = 27.59°\text{F}$$

Required temperature: *(temperature must drop from 40°F)*
$T = 40 - 27.59 = 12.41°\text{F}$ →***answer***

(b) From the figure below:



$\delta = \delta_T$

$\dfrac{PL}{AE} = \alpha\, L\, (\Delta T)$

$P = \alpha AE(T_f \,-\, T_i)$

$1200 = (6.5 \times 10^{-6})(0.25)(29 \times 10^{6})(T_f - 40)$

$T_f = 65.46^\circ \mathrm{F}$ $\rightarrow$***answer***

A bronze bar 3 m long with a cross sectional area of 320 mm^2 is placed between two rigid walls as shown in Fig. P-265. At a temperature of -20°C, the gap Δ = 25 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use α = 18.0 × 10^{-6} m/(m·°C) and E = 80 GPa.

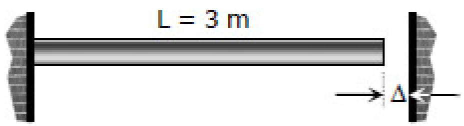
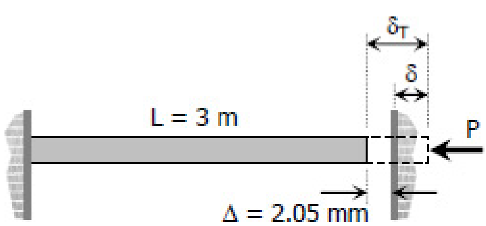


Figure P-265

Problem 265

$$\delta_T = \delta + \Delta$$



$$\alpha L\,(\Delta T) = \frac{\sigma L}{E} + 2.5$$

$$(18 \times 10^{-6})(3000)(\Delta T) = \frac{35(3000)}{80000} + 2.5$$

ΔT=487

T=487-20=467C° ***answer***

Problem 266

Calculate the increase in stress for each segment of the compound bar shown in Fig. P-266 if the temperature increases by 100°F. Assume that the supports are unyielding and that the bar is suitably braced against buckling.

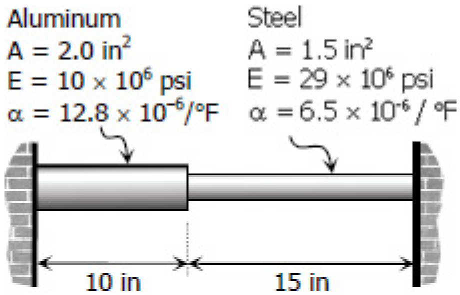
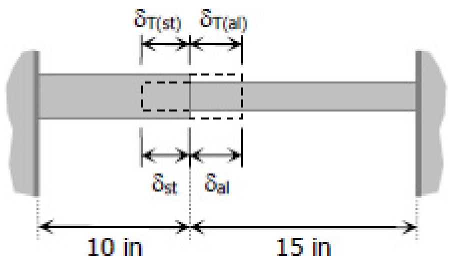


Figure P-266

Problem 266

$$\delta_T = \alpha \, L \, \Delta T$$



$$\delta_{T(st)} = (6.5 \times 10^{-6})(15)(100)$$
$$\delta_{T(st)} = 0.00975$$

$$\delta_{T(al)} = (12.8 \times 10^{-6})(10)(100)$$
$$\delta_{T(al)} = 0.0128 \text{ in}$$

$$\delta_{st} + \delta_{al} = \delta_{T(st)} + \delta_{T(al)}$$
$$\left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{al} = 0.00975 + 0.0128$$

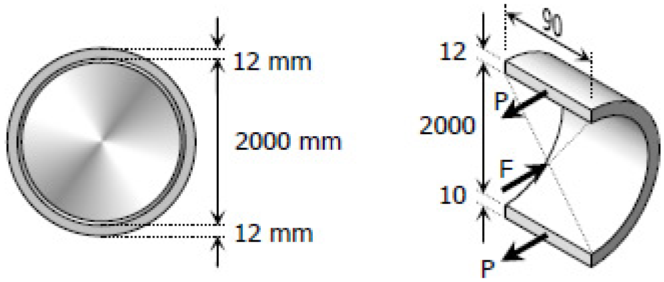
where $P = P_{st} = P_{al}$
$$\frac{P(15)}{1.5(29\times10^6)} + \frac{P(10)}{2(10\times10^6)} = 0.02255$$
$$P = 26691.84 \text{ psi}$$

$$\sigma = \frac{P}{A}$$
$$\sigma_{st} = \frac{26\,691.84}{1.5} = 17\,794.56 \text{ psi} \rightarrow$$ ***answer***
$$\sigma_{st} = \frac{26\,691.84}{2.0} = 13\,345.92 \text{ psi} \rightarrow$$ ***answer***

Problem 267

At a temperature of 80°C, a steel tire 12 mm thick and 90 mm wide that is to be shrunk onto a locomotive driving wheel 2 m in diameter just fits over the wheel, which is at a temperature of 25°C. Determine the contact pressure between the tire and wheel after the assembly cools to 25°C. Neglect the deformation of the wheel caused by the pressure of the tire. Assume α = 11.7 Î¼m/(m·°C) and E = 200 GPa.

Solution 267



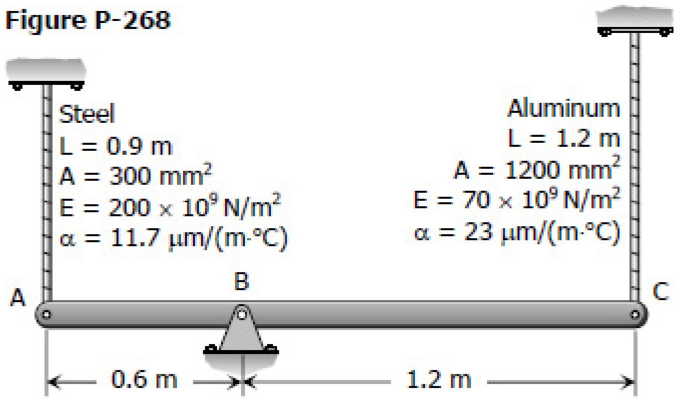
$\delta = \delta_T$
$\dfrac{PL}{AE} = \alpha\, L\, \Delta T$
$P = \alpha\, \Delta T\, AE$
$P = (11.7 \times 10^{-6})(80 - 25)(90 \times 12)(200\,000)$
$P = 138\,996\,\text{N}$

$F = 2P$
$pDL = 2P$
$p(2000)(90) = 2(138\,996)$
$p = 1.5444\,\text{MPa}$ → ***answer***

Problem 268

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.

Figure P-268



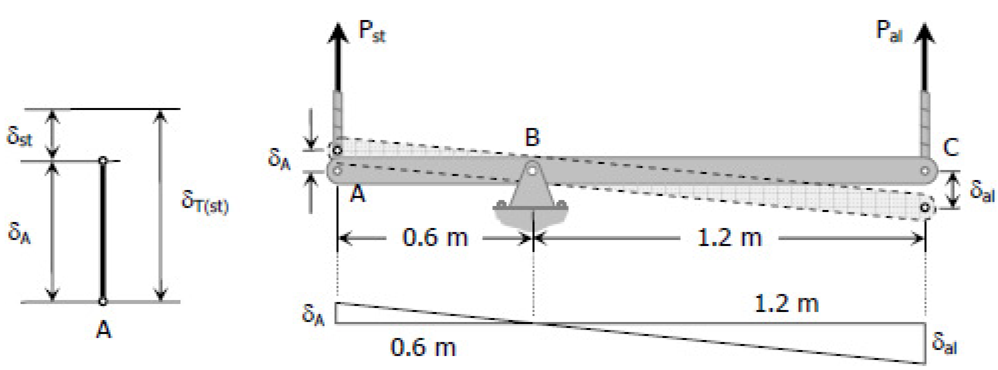
Solution 268

Contraction of steel rod, assuming complete freedom:

$$\delta_{T(st)} = \alpha L \Delta T$$
$$\delta_{T(st)} = (11.7 \times 10^{-6})(900)(40)$$
$$\delta_{T(st)} = 0.4212 \, \text{mm}$$

The steel rod cannot freely contract because of the resistance of aluminum rod. The movement of A (referred to as δ_A), therefore, is less than 0.4212 mm. In terms of aluminum, this movement is (by ratio and proportion):

$$\frac{\delta_A}{0.6} = \delta_{al} 1.2$$
$$\delta_A = 0.5 \delta_{al}$$



$$\delta_{T(st)} - \delta_{st} = 0.5\delta_{al}$$

$$0.4212 - \left(\frac{PL}{AE}\right)_{st} = 0.5\left(\frac{PL}{AE}\right)_{al}$$

$$0.4212 - \frac{P_{st}(900)}{300(200\,000)} = 0.5\left[\frac{P_{al}(1200)}{1\,200(70\,000)}\right]$$

$28080 - P_{st} = 0.4762P_{al}$ →**Equation (1)**

$\Sigma M_B = 0$
$0.6P_{st} = 1.2P_{al}$
$P_{st} = 2P_{al}$ →**Equation (2)**

Equations (1) and (2)
$28\,080 - 2P_{al} = 0.4762P_{al}$
$P_{al} = 11\,340\ \text{N}$

$$\sigma_{al} = \frac{P_{al}}{A_{al}} = \frac{11\,340}{1200}$$
$\sigma_{al} = 9.45\ \text{MPa}$ →***answer***

Problem 269

As shown in Fig. P-269, there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, Δ = 0.18 mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C. For each copper bar, A = 500 mm^2, E = 120 GPa, and α = 16.8 μm/(m·°C). For the aluminum bar, A = 400 mm^2, E = 70 GPa, and α = 23.1 μm/(m·°C).

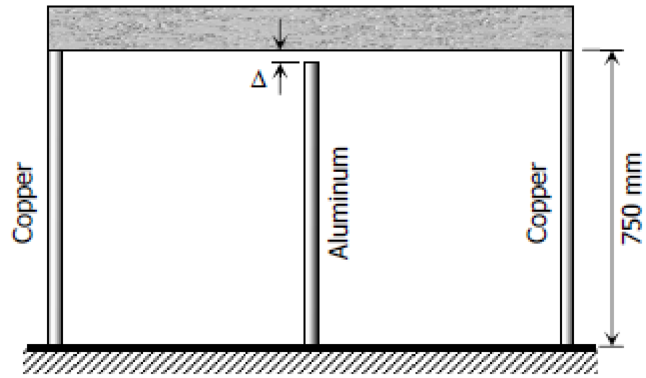
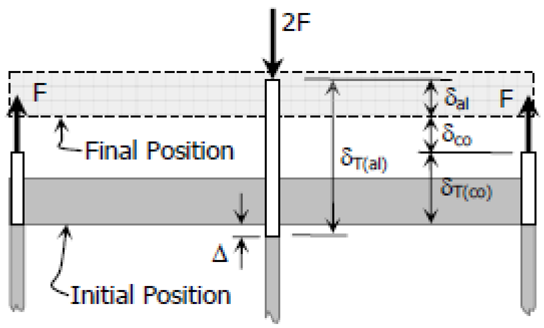


Figure P-269

Solution 269

Assuming complete freedom:

$$\delta_T = \alpha\, L\, \Delta T$$
$$\delta_{T(co)} = (16.8 \times 10^{-6})(750)(95 - 10)$$
$$\delta_{T(co)} = 1.071\ \text{mm}$$
$$\delta_{T(al)} = (23.1 \times 10^{-6})(750 - 0.18)(95 - 10)$$
$$\delta_{T(al)} = 1.472\ \text{mm}$$



From the figure:

$$\delta_{T(al)} - \delta_{al} = \delta_{T(co)} + \delta_{co}$$
$$1.472 - \left(\frac{PL}{AE}\right)_{al} = 1.071 + \left(\frac{PL}{AE}\right)_{co}$$
$$1.472 - \frac{2F(750 - 0.18)}{400(70\,000)} = 1.071 + \frac{F(750)}{500(120\,000)}$$
$$0.401 = (6.606 \times 10^{-5})\, F$$
$$F = 6070.37\ \text{N}$$

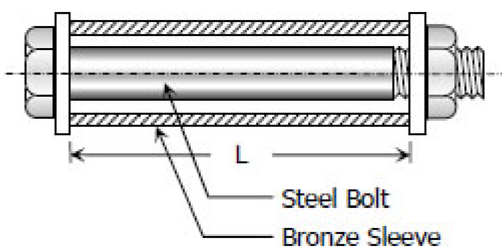
$$P_{co} = F = 6070.37 \text{ N}$$
$$P_{al} = 2F = 12\,140.74 \text{ N}$$

$$\sigma = P/A$$
$$\sigma_{co} = \frac{6070.37}{500} = 12.14 \text{ MPa} \rightarrow$$ ***answer***
$$\sigma_{al} = \frac{12\,140.74}{400} = 30.35 \text{ MPa} \rightarrow$$ ***answer***

Problem 270

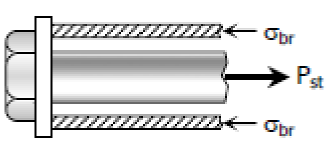
A bronze sleeve is slipped over a steel bolt and held in place by a nut that is turned to produce an initial stress of 2000 psi in the bronze. For the steel bolt, A = 0.75 in^2, E = 29 × 10^6 psi, and α = 6.5 × 10^{-6} in/(in·°F). For the bronze sleeve, A = 1.5 in^2, E = 12 × 10^6 psi and α = 10.5 × 10^{-6} in/(in·°F). After a temperature rise of 100°F, find the final stress in each material.

Solution 270



Before temperature change:

$$P_{br} = \sigma_{br} A_{br}$$
$$P_{br} = 2000(1.5)$$
$$P_{br} = 3000 \text{ lb}$$ compression



$\Sigma F_H = 0$
$P_{st} = P_{br} = 3000\,\text{lb}$ tension
$\sigma_{st} = P_{st}/A_{st} = 3000/0.75$
$\sigma_{st} = 4000\text{ psi}$ tensile stress

$\delta = \dfrac{\sigma L}{E}$
$a = \delta_{br} = \dfrac{2000L}{12 \times 10^6} = 1.67 \times 10^{-4}L$ shortening
$b = \delta_{st} = \dfrac{4000L}{29 \times 10^6} = 1.38 \times 10^{-4}L$ lengthening

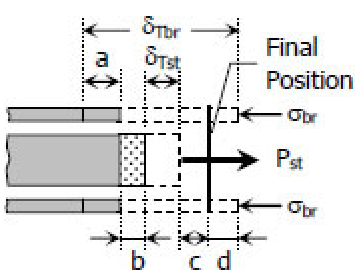
With temperature rise of 100Â°F: (*Assuming complete freedom*)
$\delta_T = \alpha\, L\, \Delta T$
$\delta_{Tbr} = (10.5 \times 10^{-6})L\,(100)$
$\delta_{Tbr} = 1.05 \times 10^{-3}L > a$

$\delta_{Tst} = (6.5 \times 10^{-6})L\,(100)$
$\delta_{Tst} = 6.5 \times 10^{-4}L$

$\delta_{Tbr} - a = 1.05 \times 10^{-3}L - 1.67 \times 10^{-4}L$
$\delta_{Tbr} - a = 8.83 \times 10^{-4}L$

$\delta_{Tst} + b = 6.5 \times 10^{-4}L + 1.38 \times 10^{-4}L$
$\delta_{Tst} + b = 7.88 \times 10^{-4}L$

$\delta_{Tbr} - a > \delta_{Tst} + b$ (see figure below)



$$\delta_{Tbr} - a - d = b + \delta_{Tst} + c$$
$$(1.05\times10^{-3}L)-(1.67\times10^{-4}L)-\left(\frac{\sigma L}{E}\right)_{br} = (1.38\times10^{-4}L)+(6.5\times10^{-4}L)+\left(\frac{PL}{AE}\right)_{st}$$
$$(8.83\times10^{-4}L) - \frac{\sigma_{br}L}{12\times10^{6}} = (7.88\times10^{-4}L) + \frac{P_{st}L}{0.75(29\times10^{6})}$$
$$9.5\times10^{-4} - \frac{P_{br}}{1.5(12\times10^{6})} = \frac{P_{st}}{0.75(29\times10^{6})}$$
$P_{st} = 20\,662.5 - 1.2083P_{br}$ → **Equation (1)**

$$\Sigma F_H = 0$$
$P_{br} = P_{st}$ → **Equation (2)**

Equations (1) and (2)

$$P_{st} = 20\,662.5 - 1.2083P_{st}$$
$$P_{st} = 9356.74\,\text{lb}$$
$$P_{br} = 9356.74\,\text{lb}$$

$$\sigma = P/A$$
$\sigma_{br} = \dfrac{9356.74}{1.5} = 6237.83\,\text{psi}$ compressive stress ***answer***

$\sigma_{st} = \dfrac{9356.74}{0.75} = 12475.66\,\text{psi}$ tensile stress ***answer***

Problem 272

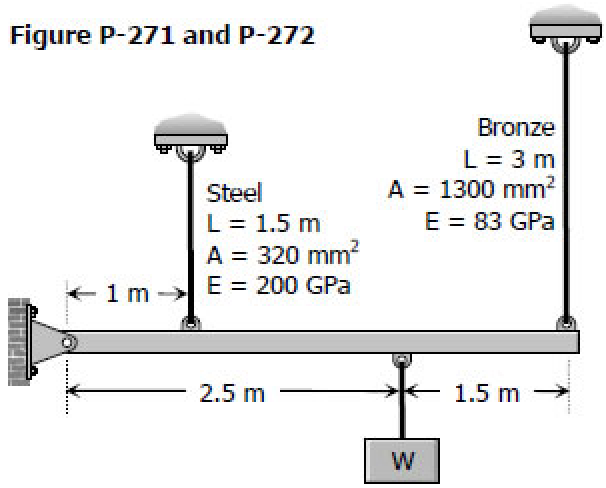
For the assembly in Fig. 271, find the stress in each rod if the temperature rises 30°C after a load W = 120 kN is applied.

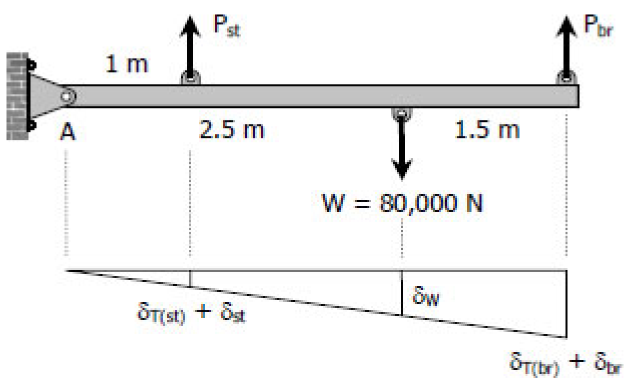
Solution 272

$$\Sigma M_A = 0$$
$$4P_{br} + P_{st} = 2.5(80\,000)$$

$$4\sigma_{br}(1300) + \sigma_{st}(320) = 2.5(80\,000)$$
$$16.25\sigma_{br} + \sigma_{st} = 625$$
$$\sigma_{st} = 625 - 16.25\sigma_{br}$$ →***Equation (1)***

Figure P-271 and P-272





$$\frac{\delta_{T(st)} + \delta_{st}}{1} = \frac{\delta_{T(br)} + \delta_{br}}{4}$$
$$\delta_{T(st)} + \delta_{st} = 0.25\left[\delta_{T(br)} + \delta_{br}\right]$$
$$(\alpha\, L\, \Delta T)_{st} + \left(\frac{\sigma L}{E}\right)_{st} = 0.25\left[(\alpha\, L\, \Delta T)_{br} + \left(\frac{\sigma L}{E}\right)_{br}\right]$$
$$(11.7{\times}10^{-6})(1500)(30) + \frac{\sigma_{st}(1500)}{200\,000} = 0.25\left[(18.9\times10^{-6})(3000)(30) + \frac{\sigma_{br}(3000)}{83\,000}\right]$$
$$0.526\,5 + 0.007\,5\sigma_{st} = 0.425\,25 + 0.009\,04\sigma_{br}$$
$$0.007\,5\sigma_{st} - 0.009\,04\sigma_{br} = -0.101\,25$$
$$0.007\,5\,(625 - 16.25\sigma_{br}) - 0.009\,04\sigma_{br} = -0.101\,25$$
$$4.687\,5 - 0.121\,875\sigma_{br} - 0.009\,04\sigma_{br} = -0.101\,25$$

$4.788\,75 = 0.130\,915\sigma_{br}$
$\sigma_{br} = 36.58deg\,;\mathrm{C}$***answer***

$\sigma_{st} = 625 - 16.25(36.58)$
$\sigma_{st} = 30.58\ \mathrm{deg;\ C}$***answer***

Problem 273

The composite bar shown in Fig. P-273 is firmly attached to unyielding supports. An axial force P = 50 kips is applied at 60°F. Compute the stress in each material at 120°F. Assume α = 6.5 × 10^{-6} in/(in·°F) for steel and 12.8 × 10^{-6} in/(in·°F) for aluminum.

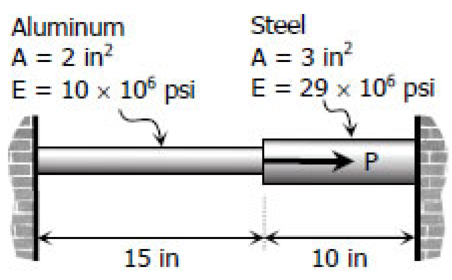
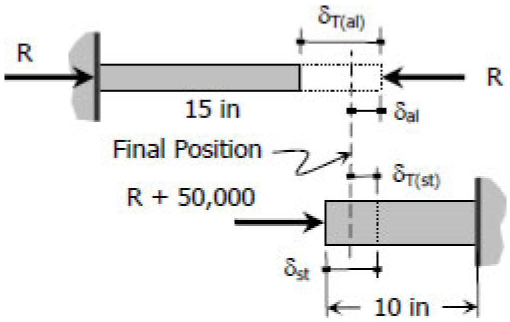


Figure P-273 and P-274

Solution 273

$$\delta_{T(al)} = (\alpha\,L\,\Delta T)_{al}$$
$$\delta_{T(al)} = (12.8 \times 10^{-6})(15)(120 - 60)$$
$$\delta_{T(al)} = 0.011\,52\,\mathrm{inch}$$



$$\delta_{T(st)} = (\alpha\, L\, \Delta T)_{st}$$
$$\delta_{T(st)} = (6.5 \times 10^{-6})(10)(120 - 60)$$
$$\delta_{T(st)} = 0.0039\,\text{inch}$$

$$\delta_{T(al)} - \delta_{al} = \delta_{st} - \delta_{T(st)}$$
$$0.011\,52 - \left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} - 0.0039$$
$$100\,224 - 6.525R = R + 50\,000 - 33\,930$$
$$84\,154 = 7.525R$$
$$R = 11\,183.25\,\text{lbs}$$

$$P_{al} = R = 11\,183.25\,\text{lbs}$$
$$P_{st} = R + 50\,000 = 61\,183.25\,\text{lbs}$$

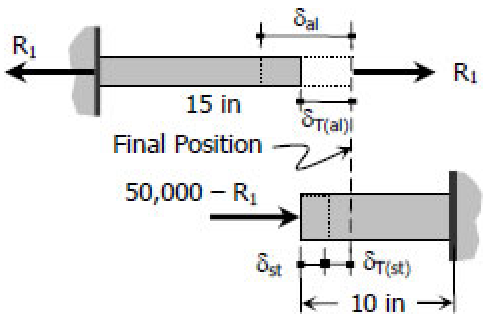
$$\sigma = \frac{P}{A}$$
$$\sigma_{al} = \frac{11\,183.25}{2} = 5\,591.62\,\text{psi}$$ ***answer***
$$\sigma_{st} = \frac{61\,183.25}{3} = 20\,394.42\,\text{psi}$$ ***answer***

Problem 274

At what temperature will the aluminum and steel segments in Prob. 273 have numerically equal stress?

Solution 274

$$\sigma_{al} = \sigma_{st}$$
$$\frac{R_1}{2} = \frac{(50\,000 - R_1)}{3}$$
$$3R_1 = 100\,000 - 2R_1$$
$$R_1 = 20\,000\,\text{lbs}$$



$$\delta = \frac{PL}{AE}$$
$$\delta_{al} = \frac{20\,000(15)}{2(10 \times 10^6)} = 0.015\,\text{inch}$$
$$\delta_{st} = \frac{(50\,000 - 20\,000)(10)}{3(29 \times 10^6)} = 0.003\,45\,\text{inch}$$

$$\delta_{al} - \delta_{T(al)} = \delta_{st} + \delta_{T(st)}$$
$$0.015 - (12.8 \times 10^{-6})(15)\,\Delta T = 0.003\,45 + (6.5 \times 10^{-6})(10)\,\Delta T$$
$$0.011\,55 = 0.000\,257\,\Delta T$$
$$\Delta T = 44.94^\circ F$$

A drop of **44.94°F** from the standard temperature will make the aluminum and steel segments equal in stress. ***answer***

CHAPTER FOUR:

SHEAR AND BENDING IN BEAMS

Problem 403

Beam loaded as shown in Fig. P-403. See the instruction.

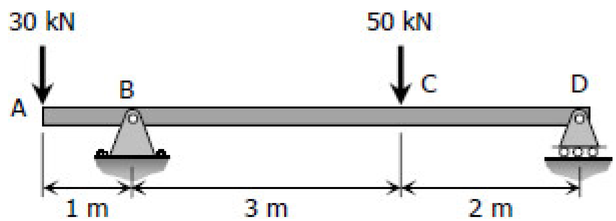


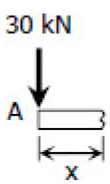
Figure P-403

Solution 403

From the load diagram:

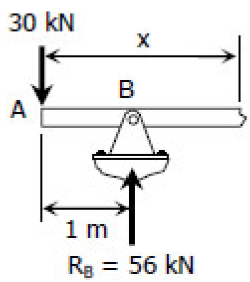
$\Sigma M_B = 0$
$5R_D + 1(30) = 3(50)$
$R_D = 24 \text{ kN}$

$\Sigma M_D = 0$
$5R_B = 2(50) + 6(30)$
$R_B = 56 \text{ kN}$



Segment AB:

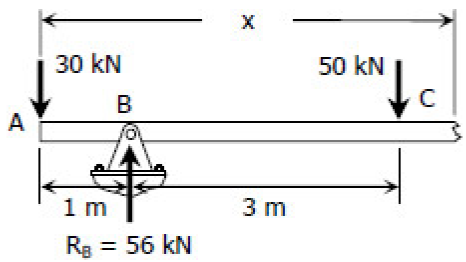
$V_{AB} = -30 \text{ kN}$
$M_{AB} = -30x \text{ kN} \cdot \text{m}$



Segment BC:

$$V_{BC} = -30 + 56$$
$$V_{BC} = 26\,\mathrm{kN}$$

$$M_{BC} = -30x + 56(x-1)$$
$$M_{BC} = 26x - 56\,\mathrm{kN\cdot m}$$

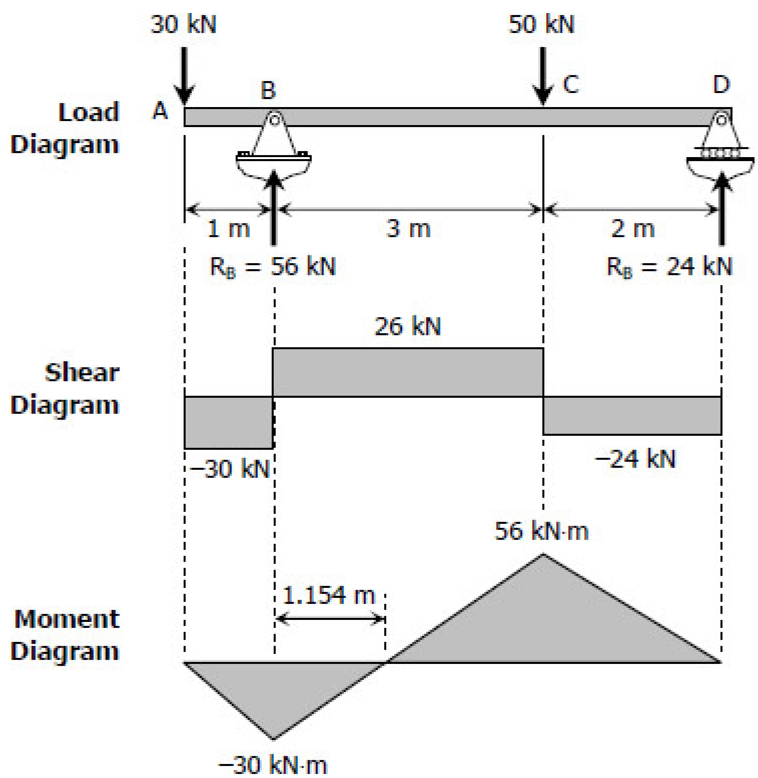


Segment CD:

$$V_{CD} = -30 + 56 - 50$$
$$V_{CD} = -24\,\mathrm{kN}$$
$$M_{CD} = -30x + 56(x-1) - 50(x-4)$$
$$M_{CD} = -30x + 56x - 56 - 50x + 200$$
$$M_{CD} = -24x + 144\,\mathrm{kN\cdot m}$$

To draw the Shear Diagram:

1. In segment AB, the shear is uniformly distributed over the segment at a magnitude of -30 kN.
2. In segment BC, the shear is uniformly distributed at a magnitude of 26 kN.
3. In segment CD, the shear is uniformly distributed at a magnitude of -24 kN.



To draw the Moment Diagram:

1. The equation $M_{AB} = -30x$ is linear, at x = 0, $M_{AB} = 0$ and at x = 1 m, $M_{AB} = -30$ kN·m.
2. $M_{BC} = 26x - 56$ is also linear. At x = 1 m, $M_{BC} = -30$ kN·m; at x = 4 m, $M_{BC} = 48$ kN·m. When $M_{BC} = 0$, x = 2.154 m, thus the moment is zero at 1.154 m from B.
3. $M_{CD} = -24x + 144$ is again linear. At x = 4 m, $M_{CD} = 48$ kN·m; at x = 6 m, $M_{CD} = 0$.

Problem 404

Beam loaded as shown in Fig. P-404. See the instruction.

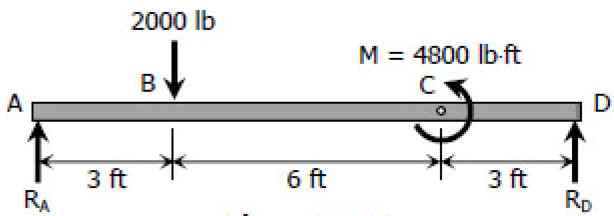
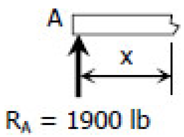


Figure P-404

Solution 404

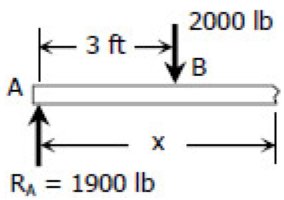
$$\Sigma M_A = 0$$
$$12R_D + 4800 = 3(2000)$$
$$R_D = 100\text{ lb}$$

$$\Sigma M_D = 0$$
$$12R_A = 9(2000) + 4800$$
$$R_A = 1900\text{ lb}$$



Segment AB:

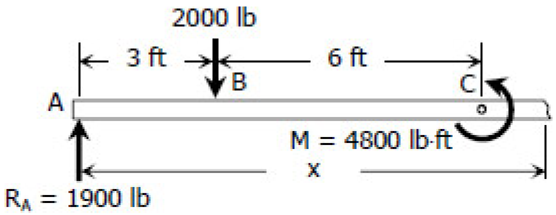
$$V_{AB} = 1900\text{ lb}$$
$$M_{AB} = 1900x\text{ lb}\cdot\text{ft}$$



Segment BC:

$$V_{BC} = 1900 - 2000$$
$$V_{BC} = -100\text{ lb}$$

$$M_{BC} = 1900x - 2000(x-3)$$
$$M_{BC} = 1900x - 2000x + 6000$$
$$M_{BC} = -100x + 6000\,\text{lb}\cdot\text{ft}$$



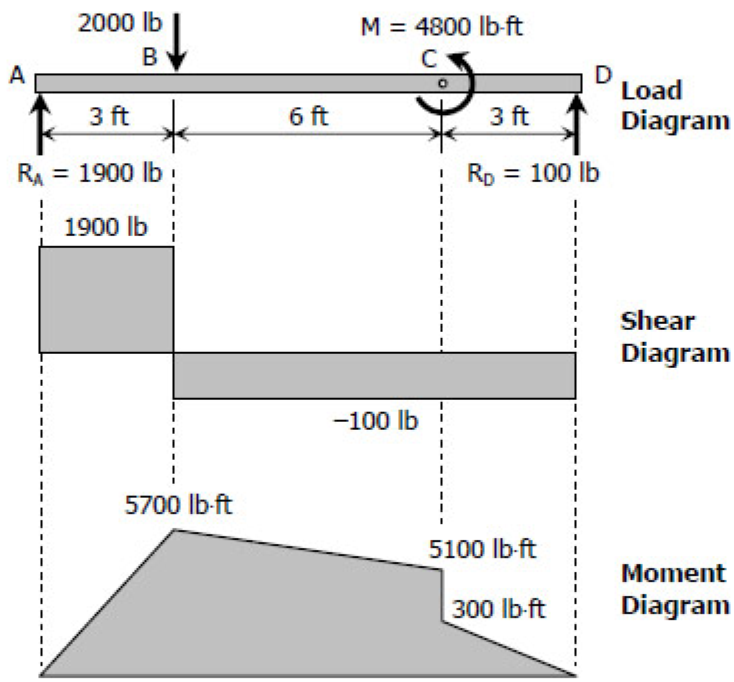
Segment CD:

$$V_{CD} = 1900 - 2000$$
$$V_{CD} = -100\,\text{lb}$$

$$M_{CD} = 1900x - 2000(x-3) - 4800$$
$$M_{CD} = 1900x - 2000x + 6000 - 4800$$
$$M_{CD} = -100x + 1200\,\text{lb}\cdot\text{ft}$$

To draw the Shear Diagram:

1. At segment AB, the shear is uniformly distributed at 1900 lb.
2. A shear of -100 lb is uniformly distributed over segments BC and CD.



To draw the Moment Diagram:

1. $M_{AB} = 1900x$ is linear; at x = 0, $M_{AB} = 0$; at x = 3 ft, $M_{AB} = 5700$ lb·ft.
2. For segment BC, $M_{BC} = -100x + 6000$ is linear; at x = 3 ft, $M_{BC} = 5700$ lb·ft; at x = 9 ft, $M_{BC} = 5100$ lb·ft.
3. $M_{CD} = -100x + 1200$ is again linear; at x = 9 ft, $M_{CD} = 300$ lb·ft; at x = 12 ft, $M_{CD} = 0$.

Problem 405

Beam loaded as shown in Fig. P-405. See the instruction.

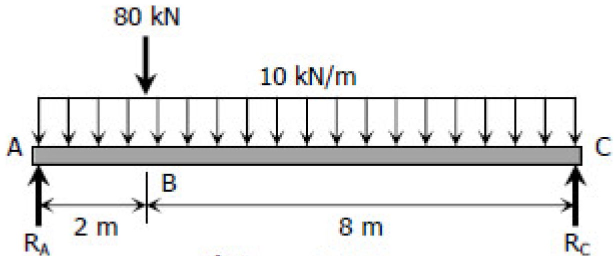
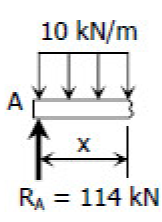


Figure P-405

Solution 405

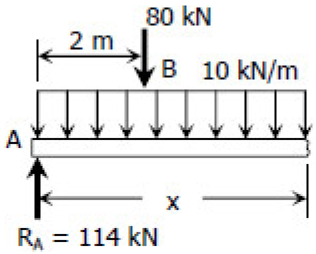
$$\Sigma M_A = 0$$
$$10R_C = 2(80) + 5[10(10)]$$
$$R_C = 66 \text{ kN}$$

$$\Sigma M_C = 0$$
$$10R_A = 8(80) + 5[10(10)]$$
$$R_A = 114 \text{ kN}$$



Segment AB:

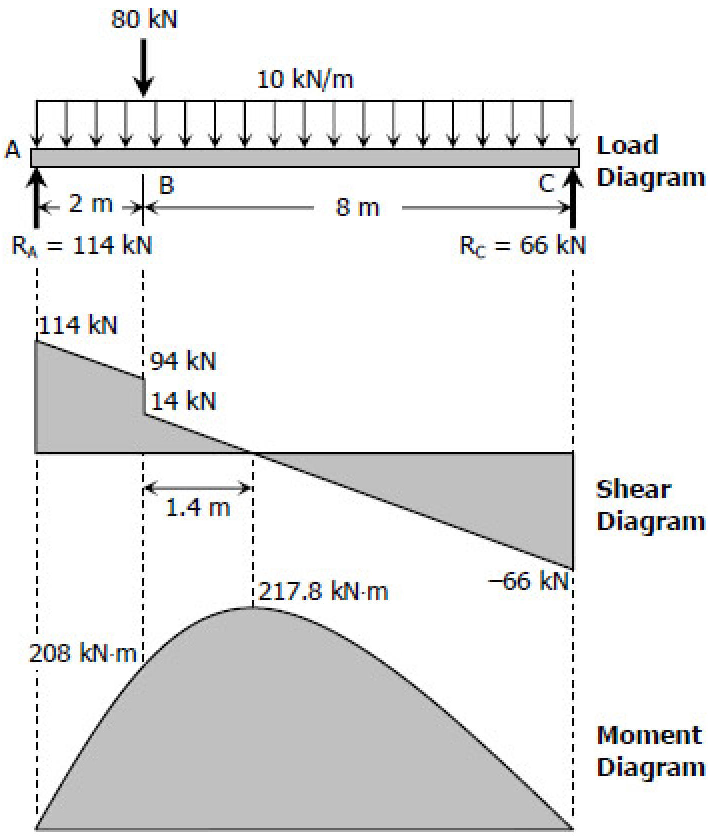
$$V_{AB} = 114 - 10x \text{ kN}$$
$$M_{AB} = 114x - 10x(x/2)$$
$$M_{AB} = 114x - 5x^2 \text{ kN} \cdot \text{m}$$



Segment BC:

$$V_{BC} = 114 - 80 - 10x$$
$$V_{BC} = 34 - 10x \text{ kN}$$
$$M_{BC} = 114x - 80(x-2) - 10x(x/2)$$
$$M_{BC} = 160 + 34x - 5x^2 \text{ kN} \cdot \text{m}$$

To draw the Shear Diagram:



1. For segment AB, $V_{AB} = 114 - 10x$ is linear; at x = 0, V_{AB} = 14 kN; at x = 2 m, V_{AB} = 94 kN.
2. $V_{BC} = 34 - 10x$ for segment BC is linear; at x = 2 m, V_{BC} = 14 kN; at x = 10 m, V_{BC} = -66 kN. When $V_{BC} = 0$, x = 3.4 m thus $V_{BC} = 0$ at 1.4 m from B.
3.

To draw the Moment Diagram:

1. $M_{AB} = 114x - 5x^2$ is a second degree curve for segment AB; at x = 0, M_{AB} = 0; at x = 2 m, M_{AB} = 208 kN·m.
2. The moment diagram is also a second degree curve for segment BC given by $M_{BC} = 160 + 34x - 5x^2$; at x = 2 m, M_{BC} = 208 kN·m; at x = 10 m, M_{BC} = 0.
3. Note that the maximum moment occurs at point of zero shear. Thus, at x = 3.4 m, M_{BC} = 217.8 kN·m.

Problem 406

Beam loaded as shown in Fig. P-406. See the instruction.

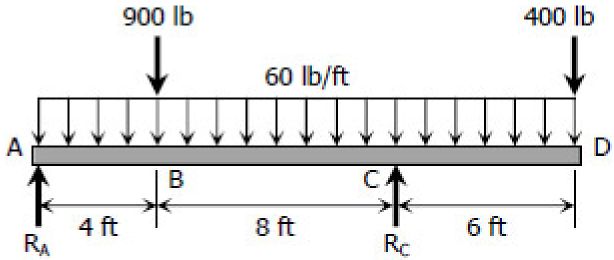
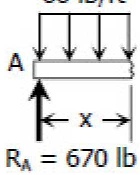


Figure P-406

Solution 406

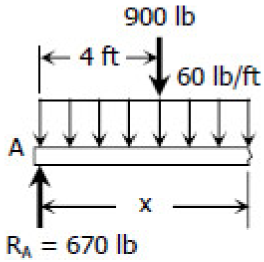
$\Sigma M_A = 0$
$12R_C = 4(900) + 18(400) + 9[(60)(18)]$
$R_C = 1710 \text{ lb}$

$\Sigma M_C = 0$
$12R_A + 6(400) = 8(900) + 3[60(18)]$
$R_A = 670 \text{ lb}$



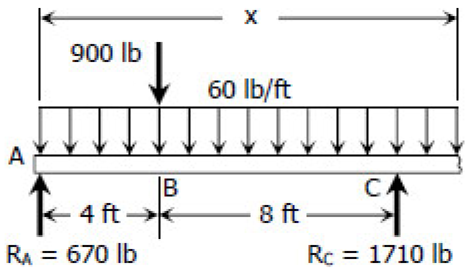
Segment AB:

$V_{AB} = 670 - 60x \text{ lb}$
$M_{AB} = 670x - 60x(x/2)$
$M_{AB} = 670x - 30x^2 \text{ lb} \cdot \text{ft}$



Segment BC:

$$V_{BC} = 670 - 900 - 60x$$
$$V_{BC} = -230 - 60x\,\text{lb}$$
$$M_{BC} = 670x - 900(x-4) - 60x(x/2)$$
$$M_{BC} = 3600 - 230x - 30x^2\,\text{lb}\cdot\text{ft}$$



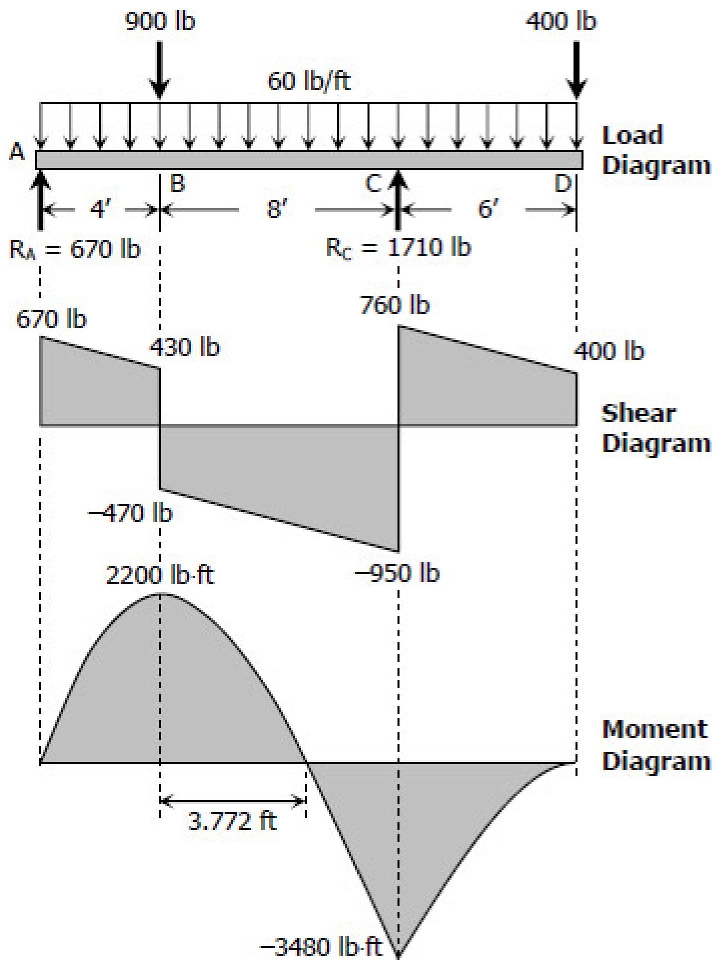
Segment CD:

$$V_{CD} = 670 + 1710 - 900 - 60x$$
$$V_{CD} = 1480 - 60x\,\text{lb}$$
$$M_{CD} = 670x + 1710(x-12) - 900(x-4) - 60x(x/2)$$

$$M_{CD} = -16920 + 1480x - 30x^2\,\text{lb}\cdot\text{ft}$$

To draw the Shear Diagram:

1. $V_{AB} = 670 - 60x$ for segment AB is linear; at $x = 0$, $V_{AB} = 670$ lb; at $x = 4$ ft, $V_{AB} = 430$ lb.
2. For segment BC, $V_{BC} = -230 - 60x$ is also linear; at $x = 4$ ft, $V_{BC} = -470$ lb, at $x = 12$ ft, $V_{BC} = -950$ lb.
3. $V_{CD} = 1480 - 60x$ for segment CD is again linear; at $x = 12$, $V_{CD} = 760$ lb; at $x = 18$ ft, $V_{CD} = 400$ lb.



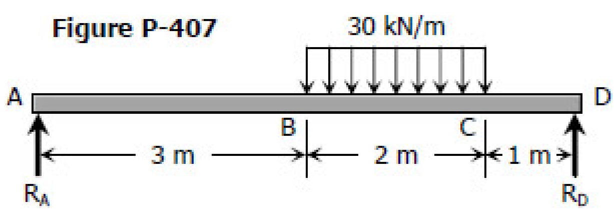
To draw the Moment Diagram:

1. $M_{AB} = 670x - 30x^2$ for segment AB is a second degree curve; at $x = 0$, $M_{AB} = 0$; at $x = 4$ ft, $M_{AB} = 2200$ lb·ft.
2. For BC, $M_{BC} = 3600 - 230x - 30x^2$, is a second degree curve; at $x = 4$ ft, $M_{BC} = 2200$ lb·ft, at $x = 12$ ft, $M_{BC} = -3480$ lb·ft; When $M_{BC} = 0$, $3600 - 230x - 30x^2 = 0$, $x = -15.439$ ft and 7.772 ft. Take $x = 7.772$ ft, thus, the moment is zero at 3.772 ft from B.

3. For segment CD, $M_{CD} = -16920 + 1480x - 30x^2$ is a second degree curve; at x = 12 ft, M_{CD} = -3480 lb·ft; at x = 18 ft, M_{CD} = 0.

Problem 407

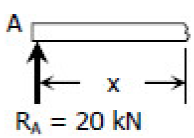
Beam loaded as shown in Fig. P-407. See the instruction.



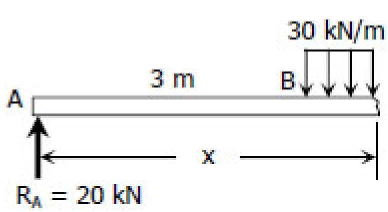
Solution 407

$\Sigma M_A = 0$
$6R_D = 4[2(30)]$
$R_D = 40\,\text{kN}$

$\Sigma M_D = 0$
$6R_A = 2[2(30)]$
$R_A = 20\text{kN}$

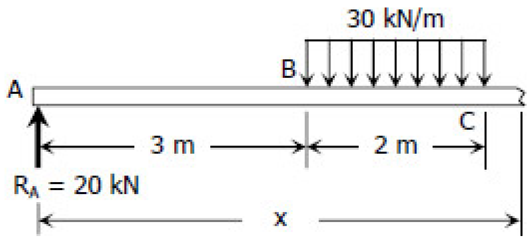


Segment AB:
$V_{AB} = 20\,\text{kN}$
$M_{AB} = 20x\,\text{kN}\cdot\text{m}$



Segment BC:
$V_{BC} = 20 - 30(x - 3)$
$V_{BC} = 110 - 30x\,\text{kN}$

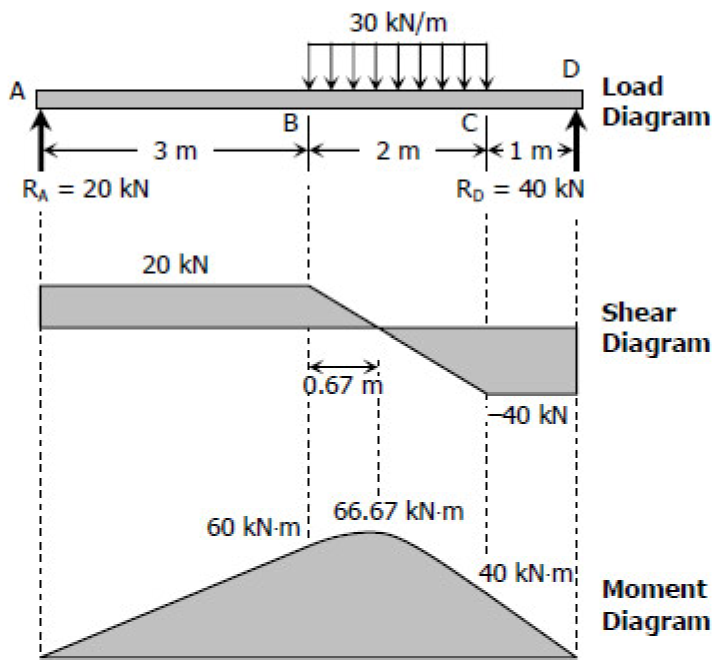
$$M_{BC} = 20x - 30(x-3)(x-3)/2$$
$$M_{BC} = 20x - 15(x-3)^2 \text{ kN}\cdot\text{m}$$



Segment CD:

$$V_{CD} = 20 - 30(2)$$
$$V_{CD} = -40 \text{ kN}$$
$$M_{CD} = 20x - 30(2)(x-4)$$
$$M_{CD} = 20x - 60(x-4) \text{ kN}\cdot\text{m}$$

To draw the Shear Diagram:



1. For segment AB, the shear is uniformly distributed at 20 kN.
2. $V_{BC} = 110 - 30x$ for segment BC; at x = 3 m, $V_{BC} = 20$ kN; at x = 5 m, $V_{BC} = -40$ kN. For $V_{BC} = 0$, x = 3.67 m or 0.67 m from B.
3. The shear for segment CD is uniformly distributed at -40 kN.

To draw the Moment Diagram:

1. For AB, $M_{AB} = 20x$; at x = 0, $M_{AB} = 0$; at x = 3 m, $M_{AB} = 60$ kN·m.
2. $M_{BC} = 20x - 15(x - 3)^2$ for segment BC is second degree curve; at x = 3 m, $M_{BC} = 60$ kN·m; at x = 5 m, $M_{BC} = 40$ kN·m. **Note:** that maximum moment occurred at zero shear; at x = 3.67 m, $M_{BC} = 66.67$ kN·m.
3. $M_{CD} = 20x - 60(x - 4)$ for segment BC is linear; at x = 5 m, $M_{CD} = 40$ kN·m; at x = 6 m, $M_{CD} = 0$.

Problem 408

Beam loaded as shown in Fig. P-408. See the instruction.

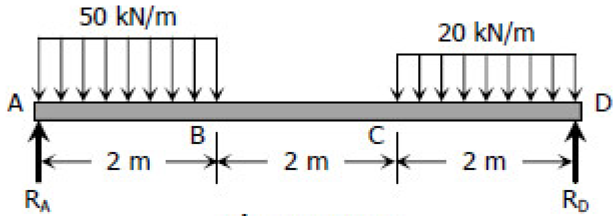
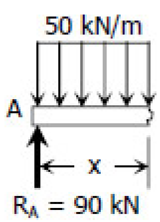


Figure P-408

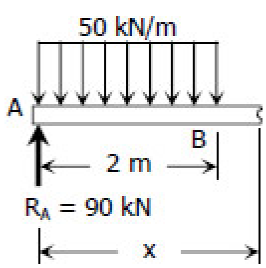
Solution 408

$\Sigma M_A = 0$
$6R_D = 1[2(50)] + 5[2(20)]$
$R_D = 50 \text{ kN}$

$\Sigma M_D = 0$
$6R_A = 5[2(50)] + 1[2(20)]$
$R_A = 90 \text{ kN}$

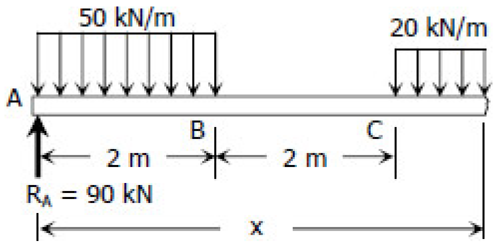


Segment AB:
$V_{AB} = 90 - 50x \text{ kN}$
$M_{AB} = 90x - 50x(x/2)$
$M_{AB} = 90x - 25x^2 \text{ kN}\cdot\text{m}$



Segment BC:
$V_{BC} = 90 - 50(2)$

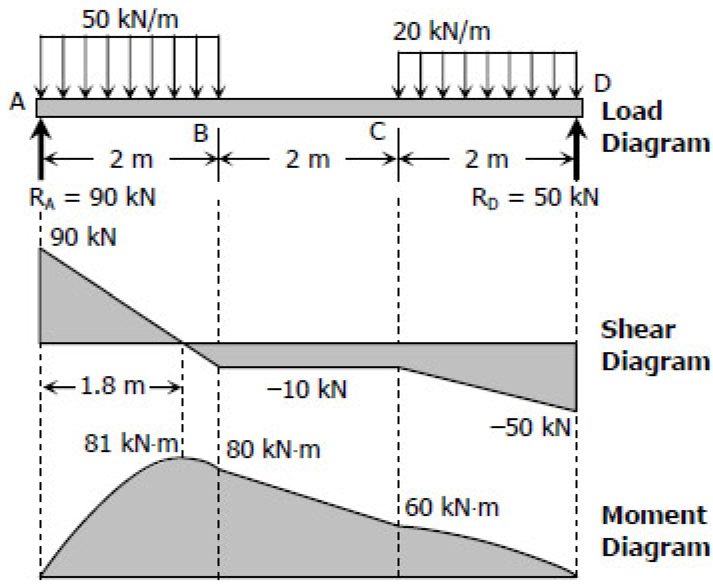
$$V_{BC} = -10\,\mathrm{kN}$$
$$M_{BC} = 90x - 2(50)(x - 1)$$
$$M_{BC} = -10x + 100\,\mathrm{kN \cdot m}$$



Segment CD:

$$V_{CD} = 90 - 2(50) - 20(x - 4)$$
$$V_{CD} = -20x + 70\,\mathrm{kN}$$
$$M_{CD} = 90x - 2(50)(x - 1) - 20(x - 4)(x - 4)/2$$
$$M_{CD} = 90x - 100(x - 1) - 10(x - 4)^2$$
$$M_{CD} = -10x^2 + 70x - 60\,\mathrm{kN \cdot m}$$

To draw the Shear Diagram:



1. V_{AB} = 90 - 50x is linear; at x = 0, V_{BC} = 90 kN; at x = 2 m, V_{BC} = -10 kN. When V_{AB} = 0, x = 1.8 m.
2. V_{BC} = -10 kN along segment BC.
3. V_{CD} = -20x + 70 is linear; at x = 4 m, V_{CD} = -10 kN; at x = 6 m, V_{CD} = -50 kN.

To draw the Moment Diagram:

1. M_{AB} = 90x - 25x^2 is second degree; at x = 0, M_{AB} = 0; at x = 1.8 m, M_{AB} = 81 kN·m; at x = 2 m, MAB = 80 kN·m.
2. M_{BC} = -10x + 100 is linear; at x = 2 m, M_{BC} = 80 kN·m; at x = 4 m, M_{BC} = 60 kN·m.
3. M_{CD} = -10x^2 + 70x - 60; at x = 4 m, M_{CD} = 60 kN·m; at x = 6 m, M_{CD} = 0.

Problem 409

Cantilever beam loaded as shown in Fig. P-409. See the instruction.

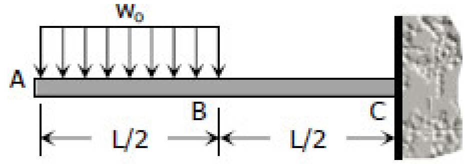
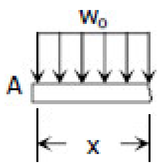


Figure P-409

Solution 409

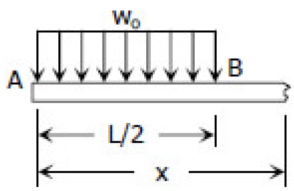


Segment AB:

$V_{AB} = -w_o x$

$M_{AB} = -w_o x(x/2)$

$MAB = -\frac{1}{2} w_o x^2$



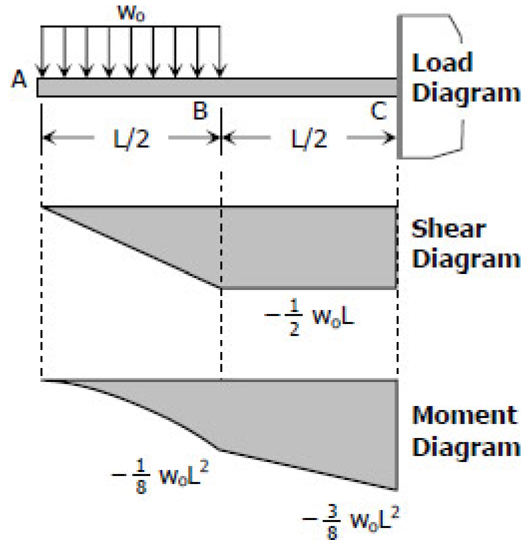
Segment BC:

$V_{BC} = -w_o (L/2)$

$V_{BC} = -\frac{1}{2} w_o L$

$M_{BC} = -w_o (L/2)(x - L/4$

$M_{BC} = -\frac{1}{2} w_o L x + \frac{1}{8} w_o L^2$



To draw the Shear Diagram:

1. $V_{AB} = -w_o x$ for segment AB is linear; at x = 0, $V_{AB} = 0$; at x = L/2, $V_{AB} = -\frac{1}{2} w_o L$.
2. At BC, the shear is uniformly distributed by $-\frac{1}{2} w_o L$.

To draw the Moment Diagram:

1. $M_{AB} = -\frac{1}{2} w_o x^2$ is a second degree curve; at x = 0, $M_{AB} = 0$; at x = L/2, $M_{AB} = -1/8\ w_o L^2$.
2. $M_{BC} = -\frac{1}{2} w_o L x + 1/8\ w_o L^2$ is a second degree; at x = L/2, $M_{BC} = -1/8\ w_o L^2$; at x = L, $M_{BC} = -3/8\ w_o L^2$.

Problem 410

Cantilever beam carrying the uniformly varying load shown in Fig. P-410. See the instruction.

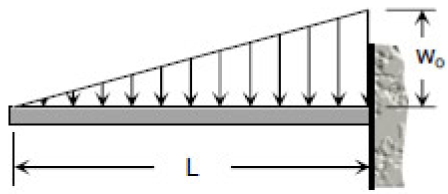
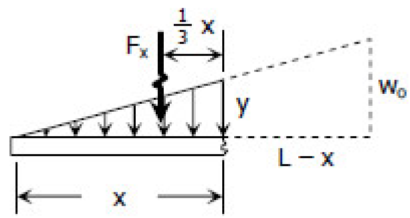


Figure P-410

Solution 410

$$\frac{y}{x} = \frac{w_o}{L}$$
$$y = \frac{w_o}{L}x$$



$$F_x = \frac{1}{2}xy$$
$$F_x = \frac{1}{2}x\left(\frac{w_o}{L}x\right)$$
$$F_x = \frac{w_o}{2L}x^2$$

Shear equation:

$$V = -\frac{w_o}{2L}x^2$$

Moment equation:

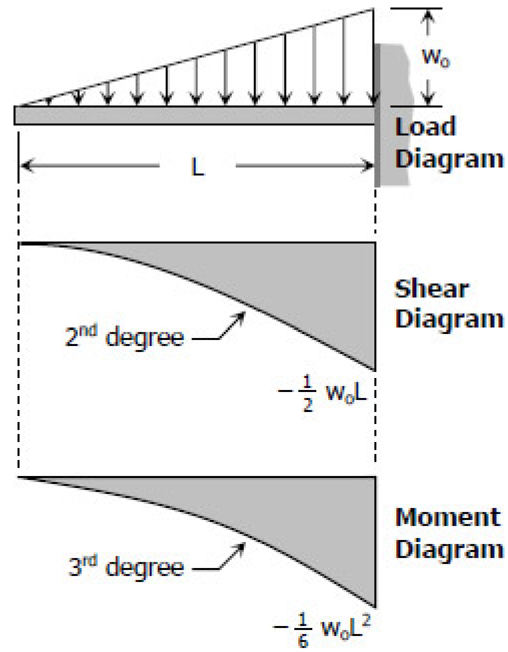
$$M = -\tfrac{1}{3}xF_x = -\frac{1}{3}x\left(\frac{w_o}{2L}x^2\right)$$
$$M = -\frac{w_o}{6L}x^3$$

To draw the Shear Diagram:

1. $V = - w_o x^2 / 2L$ is a second degree curve; at $x = 0$, $V = 0$; at $x = L$, $V = -\frac{1}{2} w_o L$.

To draw the Moment Diagram:

1. $M = - w_o x^3 / 6L$ is a third degree curve; at $x = 0$, $M = 0$; at $x = L$, $M = - 1/6\ w_o L^2$.



Problem 411

Cantilever beam carrying a distributed load with intensity varying from wo at the free end to zero at the wall, as shown in Fig. P-411. See the instruction.

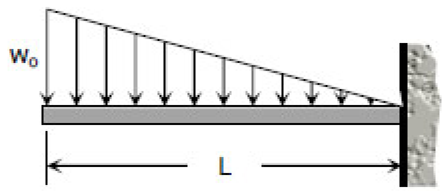
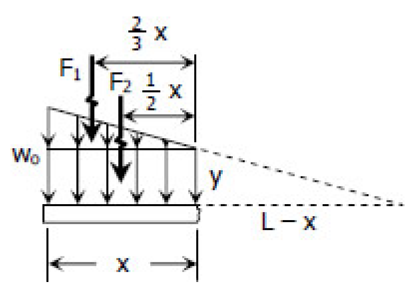


Figure P-411

Solution 411

$$\frac{y}{L-x} = \frac{w_o}{L}$$
$$y = \frac{w_o}{L}(L-x)$$

$$F_1 \tfrac{1}{2}x(w_o - y)$$
$$F_1 = \frac{1}{2}x\left[w_o - \frac{w_o}{L}(L-x)\right]$$
$$F_1 = \frac{1}{2}x\left[w_o - w_oL - \frac{w_o}{L}x\right]$$
$$F_1 = \frac{w_o}{2L}x^2$$



$$F_2 = xy = x\left[\frac{w_o}{L}(L-x)\right]$$
$$F_2 = \frac{w_o}{L}(Lx - x^2)$$

Shear equation:

$$V = -F_1 - F_2 = -\frac{w_o}{2L}x^2 - \frac{w_o}{L}(Lx - x^2)$$
$$V = -\frac{w_o}{2L}x^2 - w_o x + \frac{w_o}{L}x^2$$
$$V = \frac{w_o}{2L}x^2 - w_o x$$

Moment equation:

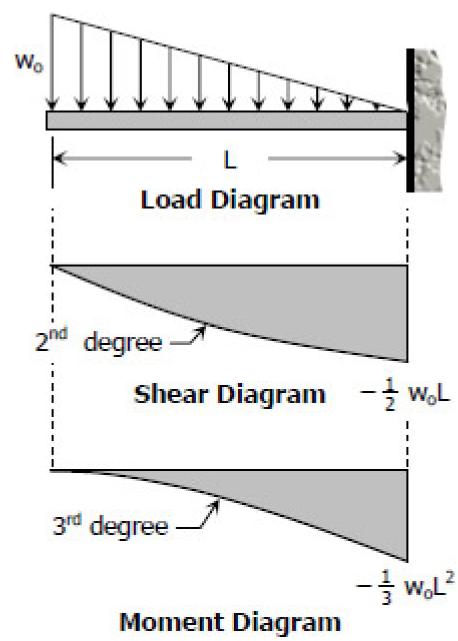
$$M = -\tfrac{2}{3}xF_1 - \tfrac{1}{2}xF_2$$
$$M = -\frac{2}{3}x\left(\frac{w_o}{2L}x^2\right) - \tfrac{1}{2}x\left[\frac{w_o}{L}(Lx - x^2)\right]$$
$$M = -\frac{w_o}{3L}x^3 - \frac{w_o}{2}x^2 + \frac{w_o}{2L}x^3$$
$$M = -\frac{w_o}{2}x^2 + \frac{w_o}{6L}x^3$$

To draw the Shear Diagram:

1. $V = w_o x^2/2L - w_o x$ is a concave upward second degree curve; at $x = 0$, $V = 0$; at $x = L$, $V = -1/2\, w_o L$.

To draw the Moment diagram:

1. $M = -w_o x^2/2 + w_o x^3/6L$ is in third degree; at $x = 0$, $M = 0$; at $x = L$, $M = -1/3\, w_o L^2$.



Problem 412

Beam loaded as shown in Fig. P-412. See the instruction.

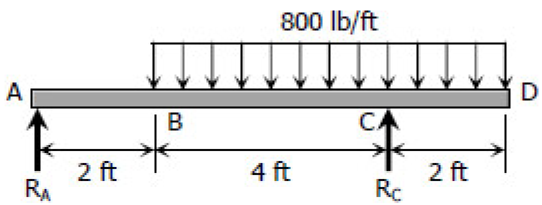
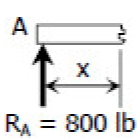


Figure P-412

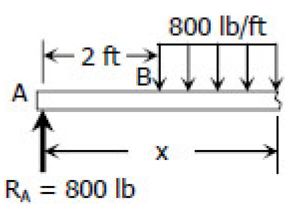
Solution 412

$\Sigma M_A = 0$
$6R_C = 5\,[\,6(800)\,]$
$R_C = 4000\,\text{lb}$

$\Sigma M_C = 0$
$6R_A = 1\,[\,6(800)\,]$
$R_A = 800\,\text{lb}$

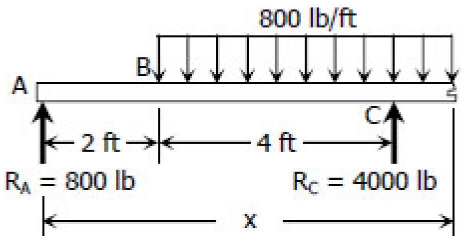


Segment AB:
$V_{AB} = 800\,\text{lb}$
$M_{AB} = 800x\,\text{lb}\cdot\text{ft}$



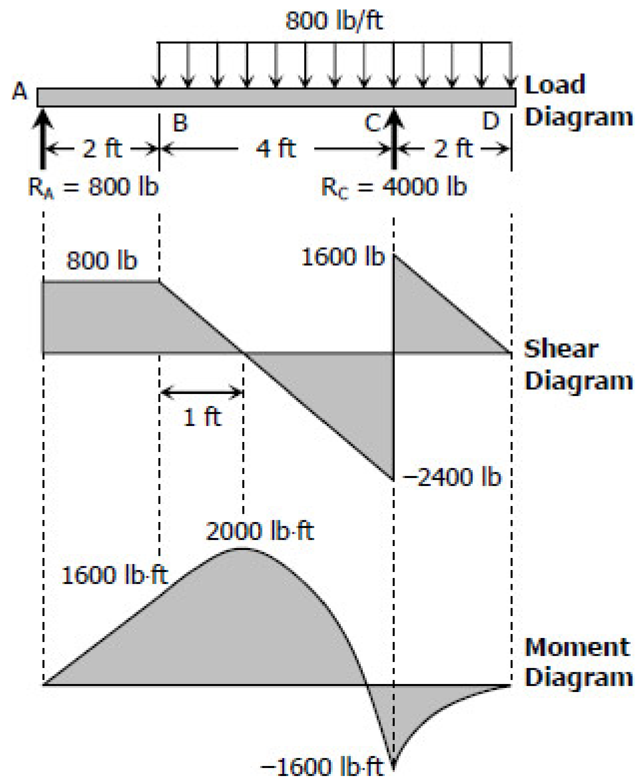
Segment BC:
$V_{BC} = 800 - 800(x-2)$
$V_{BC} = 2400 - 800x\,\text{lb}$

$$M_{BC} = 800x - 800(x-2)(x-2)/2$$
$$M_{BC} = 800x - 400(x-2)^2 \text{ lb} \cdot \text{ft}$$



Segment CD:

$$V_{CD} = 800 + 4000 - 800(x-2)$$
$$V_{CD} = 4800 - 800x + 1600$$
$$V_{CD} = 6400 - 800x \text{ lb}$$
$$M_{CD} = 800x + 4000(x-6) - 800(x-2)(x-2)/2$$
$$M_{CD} = 800x + 4000(x-6) - 400(x-2)^2 \text{ lb} \cdot \text{ft}$$



To draw the Shear Diagram:

1. 800 lb of shear force is uniformly distributed along segment AB.
2. V_{BC} = 2400 - 800x is linear; at x = 2 ft, V_{BC} = 800 lb; at x = 6 ft, V_{BC} = -2400 lb. When V_{BC} = 0, 2400 - 800x = 0, thus x = 3 ft or V_{BC} = 0 at 1 ft from B.
3. V_{CD} = 6400 - 800x is also linear; at x = 6 ft, V_{CD} = 1600 lb; at x = 8 ft, V_{BC} = 0.

To draw the Moment Diagram:

1. M_{AB} = 800x is linear; at x = 0, M_{AB} = 0; at x = 2 ft, M_{AB} = 1600 lb·ft.
2. M_{BC} = 800x - 400(x - 2)^2 is second degree curve; at x = 2 ft, M_{BC} = 1600 lb·ft; at x = 6 ft, M_{BC} = -1600 lb·ft; at x = 3 ft, M_{BC} = 2000 lb·ft.
3. M_{CD} = 800x + 4000(x - 6) - 400(x - 2)^2 is also a second degree curve; at x = 6 ft, M_{CD} = -1600 lb·ft; at x = 8 ft, MCD = 0.

Problem 413

Beam loaded as shown in Fig. P-413. See the instruction.

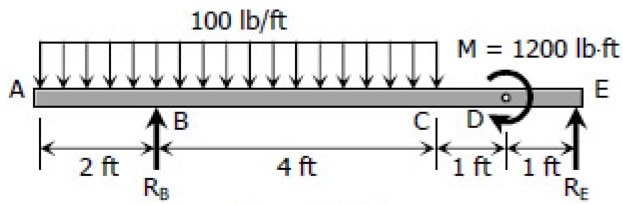
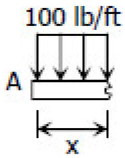


Figure P-413

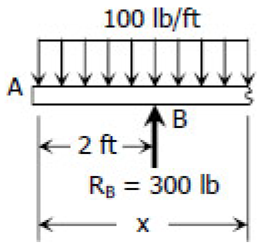
Solution 413

$\Sigma M_B = 0$
$6R_E = 1200 + 1\,[\,6(100)\,]$
$R_E = 300\,\text{lb}$

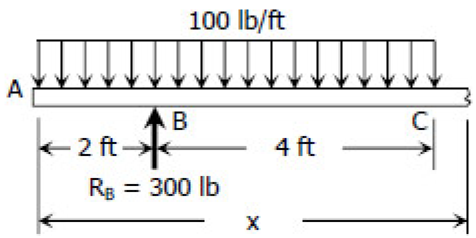
$\Sigma M_E = 0$
$6R_B + 1200 = 5\,[\,6(100)\,]$
$R_B = 300\,\text{lb}$



Segment AB:
$V_{AB} = -100x\,\text{lb}$
$M_{AB} = -100x(x/2)$
$M_{AB} = -50x^2\,\text{lb}\cdot\text{ft}$

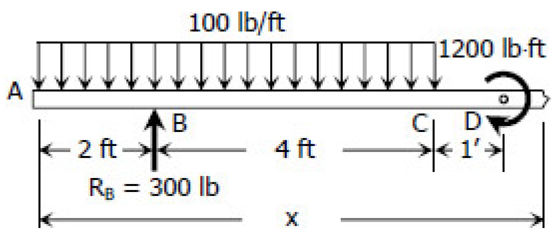


Segment BC:
$V_{BC} = -100x + 300\,\text{lb}$
$M_{BC} = -100x(x/2) + 300(x-2)$
$M_{BC} = -50x^2 + 300x - 600\,\text{lb}\cdot\text{ft}$



Segment CD:

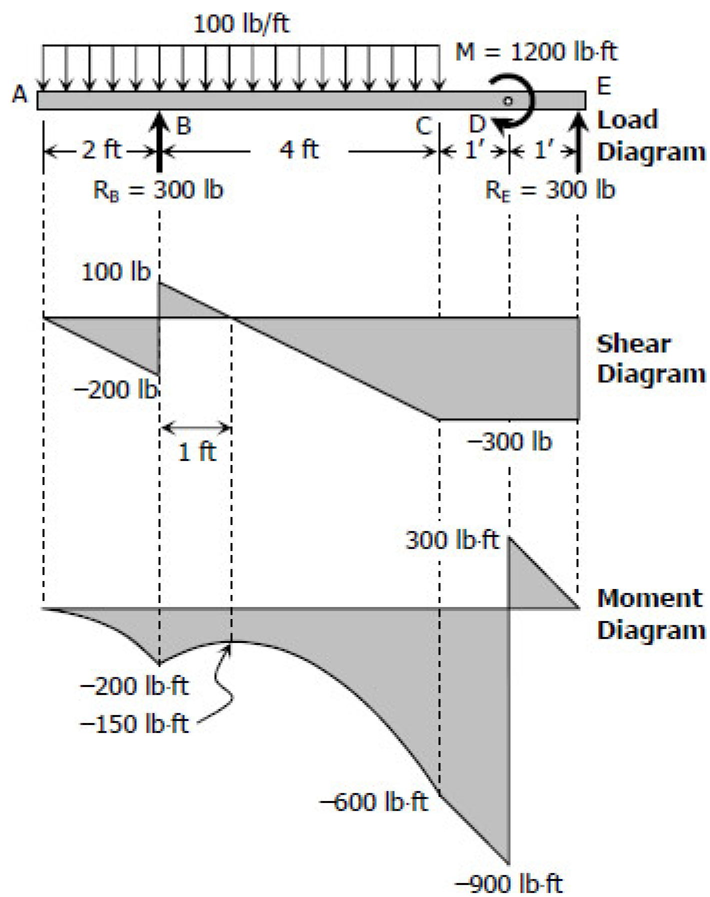
$$V_{CD} = -100(6) + 300$$
$$V_{CD} = -300\,\text{lb}$$
$$M_{CD} = -100(6)(x-3) + 300(x-2)$$
$$M_{CD} = -600x + 1800 + 300x - 600$$
$$M_{CD} = -300x + 1200\,\text{lb}\cdot\text{ft}$$



Segment DE:

$$V_{DE} = -100(6) + 300$$
$$V_{DE} = -300\,\text{lb}$$
$$M_{DE} = -100(6)(x-3) + 1200 + 300(x-2)$$
$$M_{DE} = -600x + 1800 + 1200 + 300x - 600$$
$$M_{DE} = -300x + 2400\,\text{lb}\cdot\text{ft}$$

To draw the Shear Diagram:



1. $V_{AB} = -100x$ is linear; at x = 0, $V_{AB} = 0$; at x = 2 ft, $V_{AB} = -200$ lb.
2. $V_{BC} = 300 - 100x$ is also linear; at x = 2 ft, $V_{BC} = 100$ lb; at x = 4 ft, $V_{BC} = -300$ lb. When $V_{BC} = 0$, x = 3 ft, or $V_{BC} = 0$ at 1 ft from B.
3. The shear is uniformly distributed at -300 lb along segments CD and DE.

To draw the Moment Diagram:

1. $M_{AB} = -50x^2$ is a second degree curve; at x= 0, $M_{AB} = 0$; at x = ft, $M_{AB} = -200$ lb·ft.
2. $M_{BC} = -50x^2 + 300x - 600$ is also second degree; at x = 2 ft; $M_{BC} = -200$ lb·ft; at x = 6 ft, M_{BC}

= -600 lb·ft; at x = 3 ft, M_{BC} = -150 lb·ft.

3. M_{CD} = -300x + 1200 is linear; at x = 6 ft, M_{CD} = -600 lb·ft; at x = 7 ft, M_{CD} = -900 lb·ft.
4. M_{DE} = -300x + 2400 is again linear; at x = 7 ft, M_{DE} = 300 lb·ft; at x = 8 ft, M_{DE} = 0.

Problem 414

Cantilever beam carrying the load shown in Fig. P-414. See the instruction.

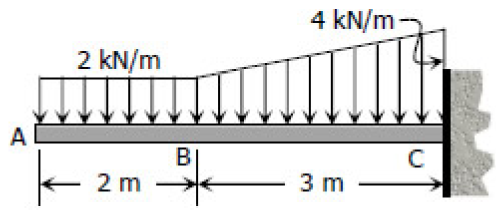
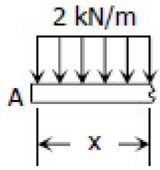


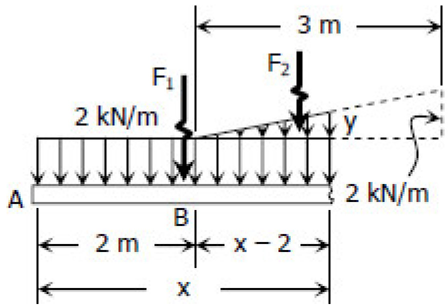
Figure P-414

Solution 414



Segment AB:

$$V_{AB} = -2x \text{ kN}$$
$$M_{AB} = -2x(x/2)$$
$$M_{AB} = -x^2 \text{ kN} \cdot \text{m}$$



Segment BC:

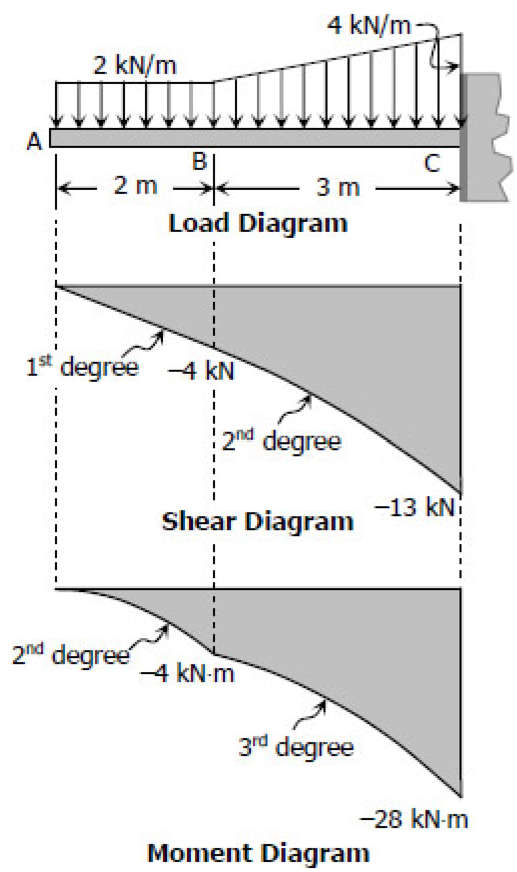
$$\frac{y}{x-2} = \frac{2}{2}$$
$$y = \frac{2}{3}(x-2)$$

$$F_1 = 2x$$

$$F_2 = \frac{1}{2}(x-2)y$$
$$F_2 = \frac{1}{2}(x-2)\left[\,\frac{2}{3}(x-2)\,\right]$$
$$F_2 = \frac{1}{3}(x-2)^2$$

$$V_{BC} = -F_1 - F_2$$
$$V_{BC} = -2x - \frac{1}{3}(x-2)^2$$

$$M_{BC} = -(x/2)F_1 - \frac{1}{3}(x-2)F_2$$
$$M_{BC} = -(x/2)(2x) - \frac{1}{3}(x-2)\left[\,\frac{1}{3}(x-2)^2\,\right]$$
$$M_{BC} = -x^2 - \frac{1}{9}(x-2)^3$$



To draw the Shear Diagram:

1. $V_{AB} = -2x$ is linear; at $x = 0$, $V_{AB} = 0$; at $x = 2$ m, $V_{AB} = -4$ kN.
2. $V_{BC} = -2x - 1/3\,(x - 2)^2$ is a second degree curve; at $x = 2$ m, $V_{BC} = -4$ kN; at $x = 5$ m; $V_{BC} = -13$ kN.

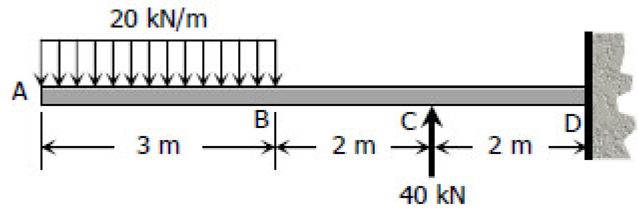
To draw the Moment Diagram:

1. $M_{AB} = -x^2$ is a second degree curve; at $x = 0$, $M_{AB} = 0$; at $x = 2$ m, $M_{AB} = -4$ kN·m.
2. $M_{BC} = -x^2 - 1/9\,(x - 2)^3$ is a third degree curve; at $x = 2$ m, $M_{BC} = -4$ kN·m; at $x = 5$ m, $M_{BC} = -28$ kN·m.

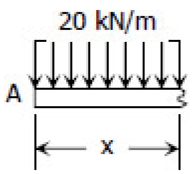
Problem 415

Cantilever beam loaded as shown in Fig. P-415. See the instruction.

Figure P-415



Solution 415

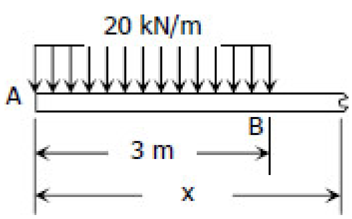


Segment AB:

$V_{AB} = -20x \text{ kN}$

$M_{AB} = -20x(x/2)$

$M_{AB} = -10x^2 \text{ kN} \cdot \text{m}$



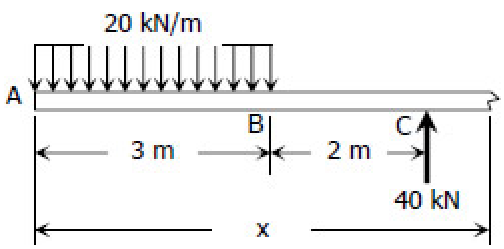
Segment BC:

$V_{BC} = -20(3)$

$V_{BC} = -60 \text{ kN}$

$M_{BC} = -20(3)(x - 1.5)$

$M_{BC} = -60(x - 1.5) \text{ kN} \cdot \text{m}$



Segment CD:

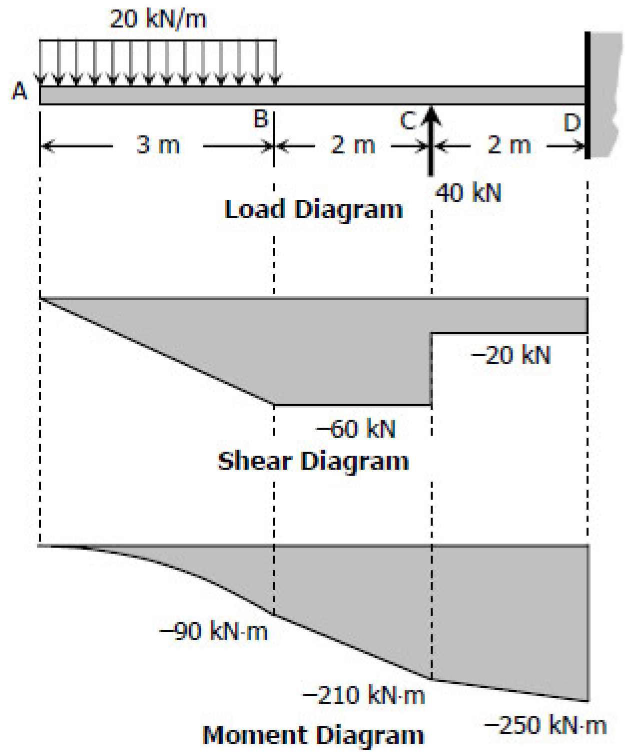
$V_{CD} = -20(3) + 40$

$V_{CD} = -20 \text{ kN}$

$$M_{CD} = -20(3)(x - 1.5) + 40(x - 5)$$
$$M_{CD} = -60(x - 1.5) + 40(x - 5)\ \mathrm{kN \cdot m}$$

To draw the Shear Diagram

1. V_{AB} = -20x for segment AB is linear; at x = 0, V = 0; at x = 3 m, V = -60 kN.
2. V_{BC} = -60 kN is uniformly distributed along segment BC.
3. Shear is uniform along segment CD at -20 kN.



To draw the Moment Diagram

1. $M_{AB} = -10x^2$ for segment AB is second degree curve; at x = 0, M_{AB} = 0; at x = 3 m, M_{AB} = -90 kN·m.
2. M_{BC} = -60(x - 1.5) for segment BC is linear; at x = 3 m, MBC = -90 kN·m; at x = 5 m, M_{BC} = -210 kN·m.
3. M_{CD} = -60(x - 1.5) + 40(x - 5) for segment CD is also linear; at x = 5 m, M_{CD} = -210 kN·m, at x = 7 m, M_{CD} = -250 kN·m.